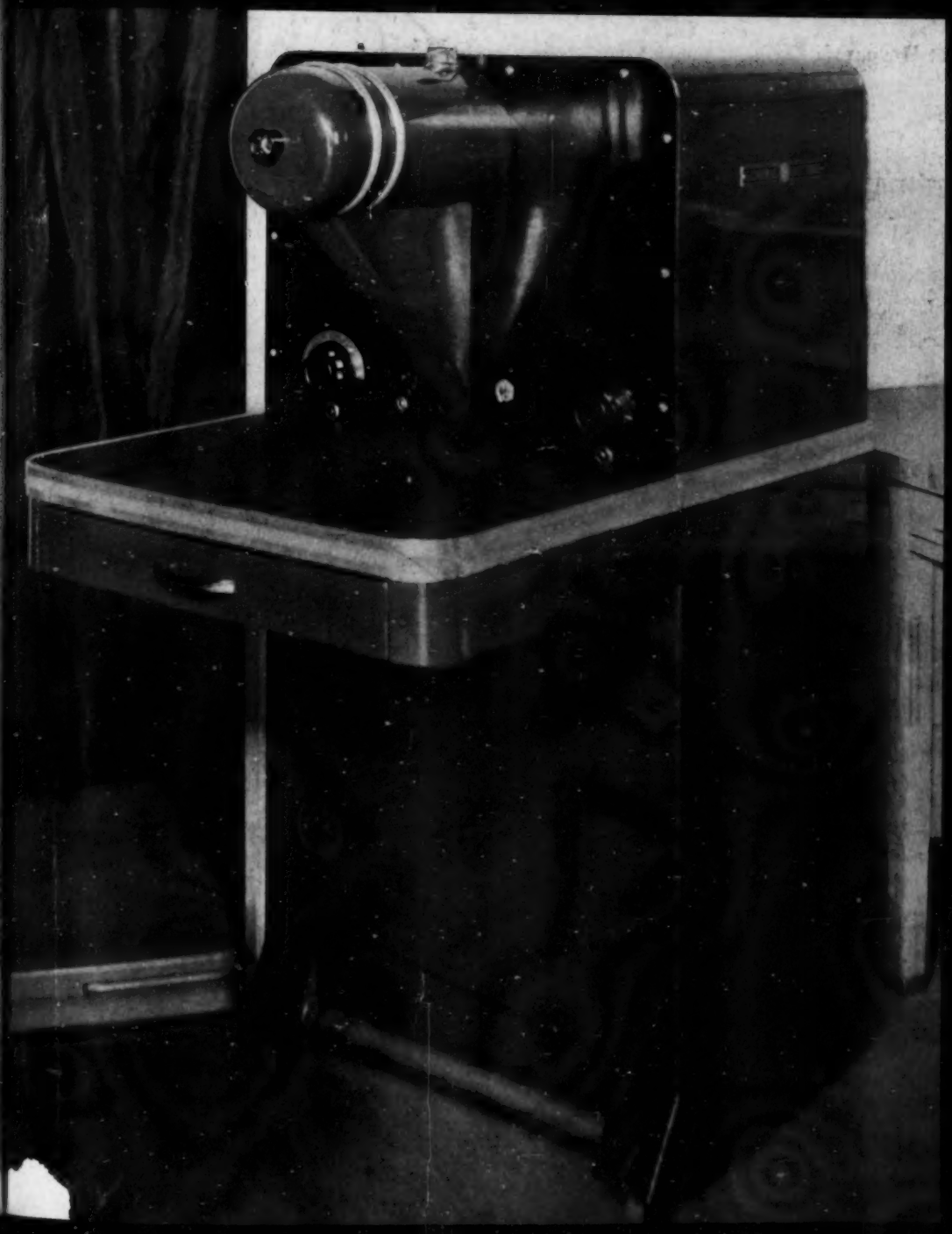
JOURNAL OF APPLIED PHYSICS



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Electrostatic Electron Microscopy. III

By C. H. Bachman and Simon Ramo

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This article, the final one of a series on the design of electrostatic electron microscopes, contains a description of an instrument which illustrates the principles previously discussed. The microscope described is believed to be the first constructed with the object of providing the greatest of simplicity in construction, operation, and maintenance with the design parameters balanced to give a particular range of resolving power. The range chosen is about ten times the light microscope. The instrument is permanently aligned and utilizes external photography. The over-all size and weight of the instrument, as well as the number and complexity of components, are materially less than previously described instruments.

INTRODUCTION

THE previous articles of this series[1,2] have pointed out some of the factors to be considered in designing an electron microscope with electrostatic electron-optical components. These principles will now be illustrated by the description of a sample instrument. This instrument was constructed, for one thing, as a test of the usefulness of the electrostatic lens approach for a large portion of the field of electron microscope application. However, there was another and equally important objective which led to the choice of the design parameters. It was to construct an instrument which would give a particular resolving power range with the maximum of simplicity in manufacture, operation, and maintenance.

The range chosen was about ten times superior to the light microscope or a resolving power of about 200A. This was regarded as a field of microscopy of tremendous importance and one whose secrets are not more likely to be discovered quickly by a wider range instrument (which entails many complexities in achieving its performance) than by a radically simplified instrument whose operation and construction are everywhere balanced for this particular range.

Unlike previously described electron microscopes, no element of simplification was ruled out because it might affect the resolving power. Simplification and resolving power were balanced in an effort to have no part of the instrument "over-designed."

CHIEF DESIGN CHARACTERISTICS

The resulting instrument has the following basic characteristics of design:

(1) The electrons are accelerated and focused by a single, unregulated voltage.

(2) An imaging system of three electrostatic lenses is built into one compact cartridge. The total electron source to fluorescent screen distance is only eleven inches.

[1] C. H. Bachman and Simon Ramo, J. App. Phys. **14**, 8 and 69 (1943).
[2] C. H. Bachman and Simon Ramo, Phys. Rev. **62**, 494 (1942).

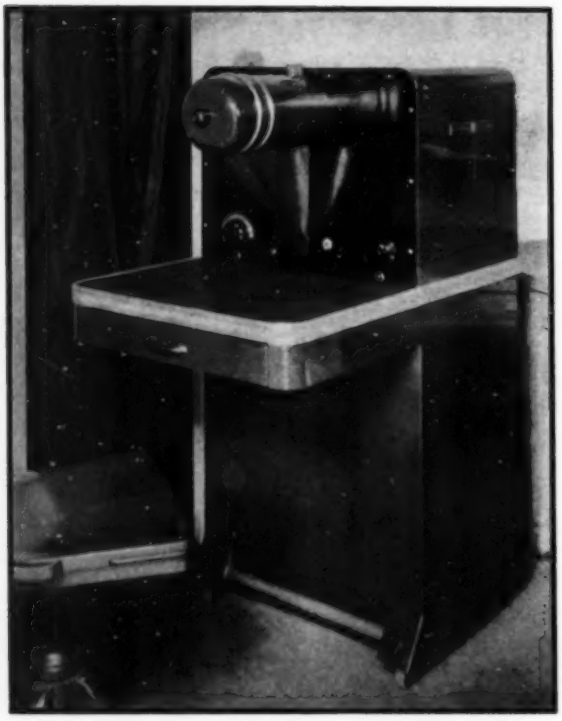

FIG. 1. The General Electric simplified electron microscope.

(3) The electron lenses are of unvarying focal length. Variation of magnification is obtained by conventional interchange of glass light-optical lenses which form the last magnification stage of the instrument. If desired, the electronic magnification may be shifted in range by the removal of one electron lens or the variation of electron lens positions.

(4) The short electron path resulting from the use of a multiple lens electron-optical system in combination with a stage of light-optical magnification makes it possible to align the lenses permanently.

(5) The above factors result in an unusually small electronic chamber. It can be re-evacuated to operating conditions in about two minutes after insertion of a new specimen. This, and the external photography technique, which a final light-optics stage facilitates, eliminate the need for specimen and photographic film insertion locks. Change of voltage or filament current or replacement of the filament itself requires no attention of any kind to the electron-optical system.

(6) The vacuum system consists of a single, small, rough-vacuum pump and an air-cooled, oil-diffusion, high vacuum pump. No special facilities are required and the instrument can be operated wherever 110-volts, 60-cycle power is available.

(7) The aforementioned characteristics result in a relatively small instrument both in weight and physical dimensions. The microscope is mounted on castors and can easily be moved from place to place.

MICROSCOPE COMPONENTS

Figure 1 shows the simplified microscope as it appears ready for visual observation of specimens. Figure 2 shows in cross section the arrangement of components. The electronic chamber is mounted horizontally above the desk-like portion of the instrument so that an operator seated before the instrument will find the fluorescent screen image conveniently located before his eyes. The controls are at his finger tips; they permit adjustment of accelerating voltage, brightness (filament current), specimen position, and connection of the chamber with the vacuum pumping system. The electronic chamber will be discussed in more detail later.

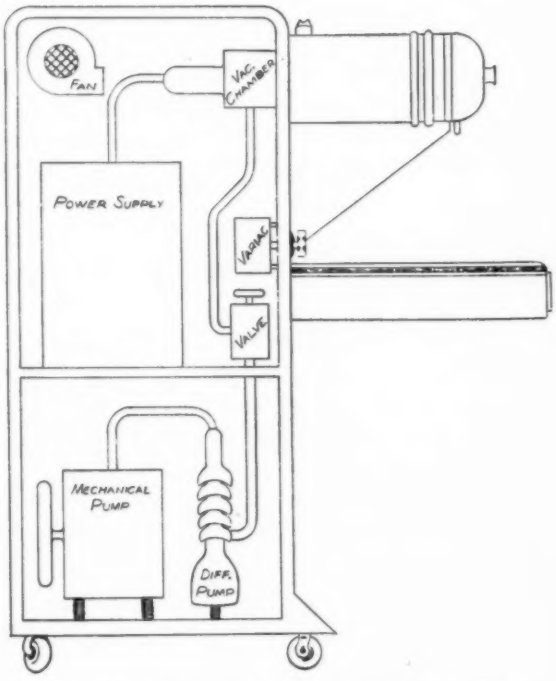

FIG. 2. Arrangement of microscope components.

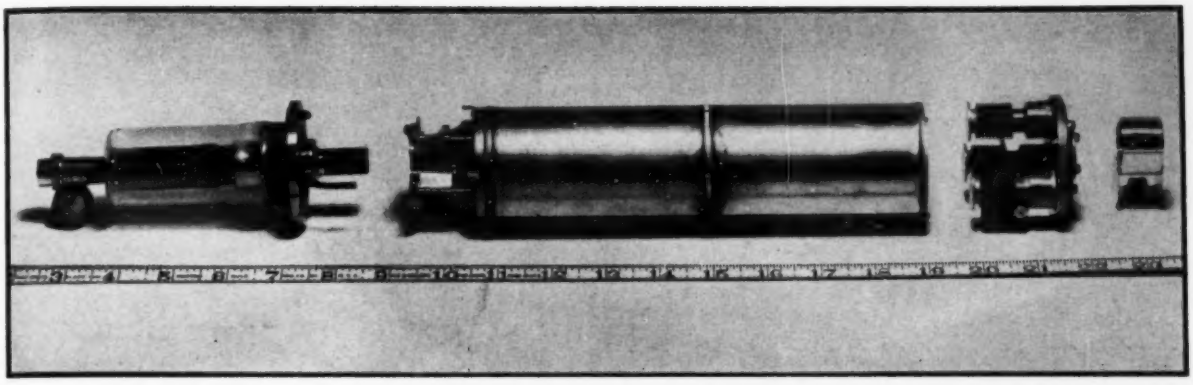

FIG. 3. Electron-optical components.

The electronic chamber is connected to the oil diffusion pump by copper tubing through the vacuum valve. The conventional method of maintaining the high vacuum required in the microscope calls for continuous operation of two pumps in series. Some form of "diffusion" pump is ordinarily used to reach the "fine" vacuum and this is "backed up" with a mechanical pump which furnishes the "rough" vacuum. Both types of pumps are available in a wide range of capacities. Of the two common types of diffusion pumps, oil and mercury, the oil pump is better suited to the microscope application since back diffusion of mercury vapor into the evacuated chamber (a phenomenon inherent in all diffusion pumps) may result in injurious reactions. Also low vapor pressure oils are available which make unnecessary the use of cold traps and water cooling required by mercury pumps.

The fore pump used is a Welch Duo Seal, 21 liters per minute capacity. It has been mounted on a vibration proof suspension. This pump backs up a two-stage oil diffusion pump of Distillation Products, Inc., design. This type was selected because of its fractionating column, self-cleaning feature, and because it lends itself to air cooling. The Cenco vacuum valve used contains no rubber gaskets, the seating arrangement being a hardened cone which is pushed against an edged brass seat. There is an offset in the glass tube between pump and valve which aids in condensing any back diffused oil vapors.

The power supply consists of a conventional half-wave rectifier and a 3-volt transformer for heating the microscope filament and is completely self-contained in the oil tank. A two-wire, shielded cable brings the filament supply to the gun connectors at the high voltage negative to ground. High potential and filament current are each controlled by a Variac in the transformer primary. Voltage can be varied smoothly from a very low value up to about 35 kv.

ELECTRON-OPTICAL COMPONENTS

Figures 3 and 4 show further details of construction of the electron-optical components of

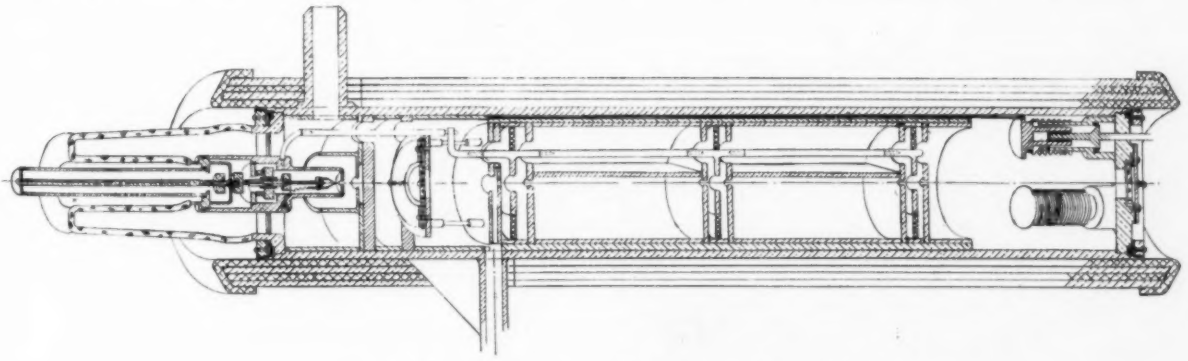

FIG. 4. Cross-sectional sketch of electronic chamber.

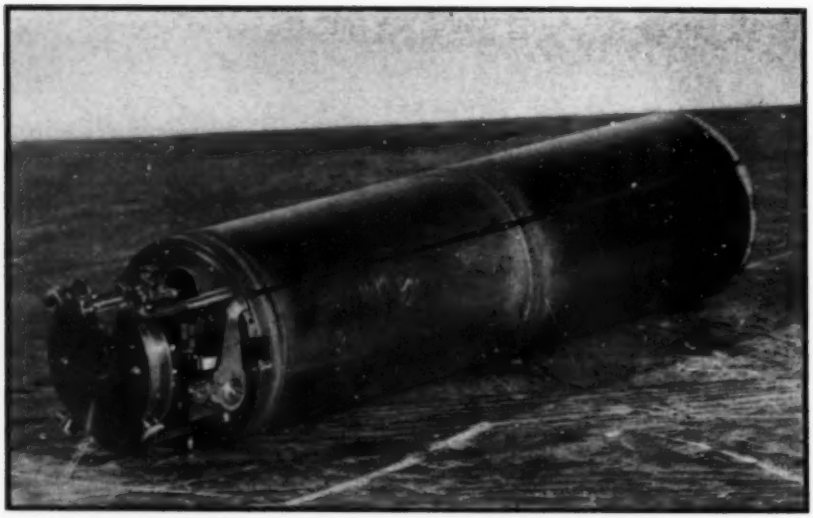

FIG. 5. Specimen manipulator and lens cartridge.

the instrument. The electron gun is a cylindrical shell containing a filament holder which can be adjusted for centering and spacing with respect to the microscope axis. The end cap screws off and the filament can then be replaced easily. This gun shell screws onto a thimble which is sealed to the re-entrant glass insulator, the other end of this being sealed to a ring which seats in the end of the vacuum chamber. The hairpin tungsten filament can be adjusted with respect to the aperture in the end cap by means of side screws and heating current is brought to it by a co-axial arrangement of tubing and rod as shown. A polished anode cup follows next along the axis and this serves to draw the electrons from the filament, through the end cap aperture; some of them pass through the aperture in the anode and move down the microscope axis at anode velocity. Magnetic shielding disks are placed following the anode to shield the magnetic field due to the filament current from the specimen region.

Before reaching the specimen, the beam is further limited by means of an aperture in a molybdenum disk mounted on the manipulator. This aperture is of such diameter that it subtends a solid angle of about 10^{-5} radian at the sample and it is so arranged that it can be adjusted for careful centering on the axis of the lens units. This adjustable aperture, the specimen manipulator, and the lens system are all part of a compact unit (Fig. 5) which can be assembled and checked for alignment before insertion in the vacuum chamber.

The manipulator provides three directions of motion, all at right angles, and each independent of the others. Two are used for scanning the object, the third for moving it into best (focused) position in front of the fixed focal length lenses. The motions are carried by three sets of ways provided with miniature balls and retainer pins.

FIG. 6. Specimen holder and insertion rod.

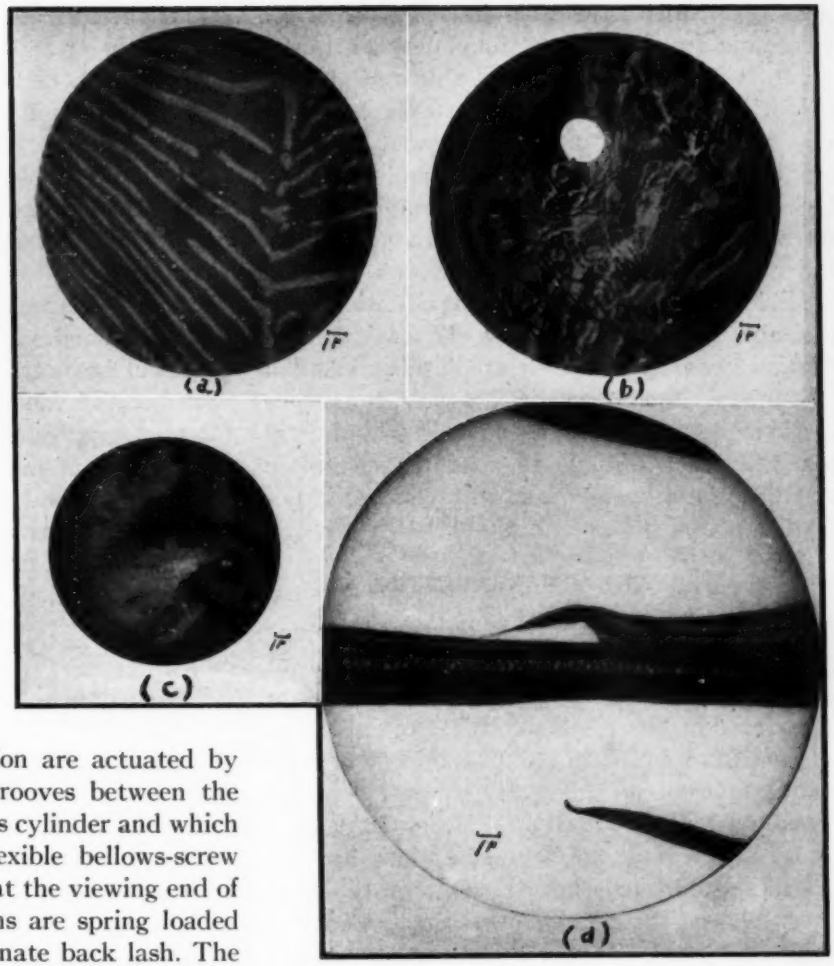

FIG. 7. Sample micrographs: (a) Formvar replica of metallographic specimen of carbonyl iron; (b) Formvar replica of metallographic specimen of 1-1 carbon steel, oil quenched; (c) recrystallized NaCl; (d) detail of fringe along edge of mosquito wing.

Levers controlling the motion are actuated by push rods which slide in grooves between the vacuum cylinder and the lens cylinder and which are, in turn, pushed by flexible bellows-screw devices set in the end plate at the viewing end of the microscope. The motions are spring loaded at the manipulator to eliminate back lash. The lever-screw combination gives to the scanning motion a total ratio about equivalent to 300 TPI screw. The focus motion has about half this value.

The object to be viewed (assumed to be suspended, in the conventional electron microscope manner, in holes of a small disk of fine mesh metal) is first mounted on a holder whose handle is several inches long (Fig. 6). The object holder is then inserted through the hole in the top of the vacuum chamber until it slips into the carrier provided on the manipulator where it is held snugly by a compression spring. The handle is allowed to rest on the side of the insertion tube and does not restrict the object as it is moved about.

Each electron lens consists of three disks with rounded apertures. The center disk is insulated and held at the cathode potential by means of a side rod assembly (Fig. 4) which leads from the gun along one side of the vacuum chamber through holes in the various other elements in the chamber and touches the outer edge of each center electrode. This side rod system must be kept sufficiently far from the axis so as not to introduce substantial cross electric fields in the lens. All lens plates are highly polished and are held in good alignment by the fit of their outside diameters in the containing cylinder. Spacing along the axis is accomplished by cylindrical spacers, and in assembling the unit, disks and spacers are stacked in the proper order in the containing cylinder. Though in practice one will probably most often choose to vary magnification by changing the external, light-optics stage, the electronic magnification is easily varied by changing spacers. Such a variation in electronic magnification, since it occurs ahead of the fluorescent screen, will probably serve in practice mainly to shift resolving power range (see below).

The fluorescent screen is deposited on a small

glass disk which is held on the end plate of the electronic chamber by a seat and gasket arrangement. In order to achieve desirable optical magnification of the image without introducing limits due to particle size of the fluorescent material, pin holes, and diffusion of light and electrons, techniques for laying thin and unusually uniform screens have been worked out. In general, in order to make the exposure time as short as possible for a given over-all magnification, it is desirable to choose the lowest electronic magnification possible. If the electronic magnification falls too low, the screen will, however, commence to limit resolving power.[1] This is why it will sometimes be desirable to alter the electronic magnification in accordance with changes in desired resolving power.

OPERATION OF INSTRUMENT

Though the limits of this instrument have not yet been completely determined, a number of micrographs have been taken, a few of which are reproduced in Fig. 7. All recordings to date have been made with about from 3000 to 7000 or 8000 diameters total magnification, of which about seven times was in the last (light-optics) stage. Under these conditions, a quick appraisal of the general structure of the specimen is possible by direct viewing of a large field directly on the fluorescent screen at magnifications often used in a light microscope. Any particular area may then be examined at several-fold more magnification by the insertion of a wide field eyepiece. This light-optical arrangement is a very efficient light gatherer and with the flat field of the eyepiece offers little chance of eye strain when used over long periods.

For photographing, the eyepiece is slipped from its holder and a simple camera is substituted. The photographs of Fig. 7 were taken with a simple camera using a single lens (Bausch and Lomb special design). Focus on the fluorescent screen can be pre-set and the camera locked in place if desired.

It will be noted that no special pre-exhaust chamber and vacuum locks are necessary either for photography or specimen replacement. The use of external photography completely removes one problem; and in the case of specimen insertion, the volume of the whole chamber is so small that a period of only three minutes is necessary to change specimens and be back in operation.

CONCLUSIONS

The extreme simplicity of design and construction would lead one to expect almost complete absence of care on the part of the operator. Such has indeed been the case. The limits of the general technique—unregulated voltage; multistage electrostatic lenses acting as a "booster" for a light-optics stage; external photography—are not yet known. However, the general performance of this sample instrument has led the authors to believe that the technique is well suited to the range of electron microscopy for which the instrument was designed.

The Evolution of Color Vision

BY GORDON L. WALLS

Bausch and Lomb Optical Company, Rochester, New York

COLOR vision is something we tend to take for granted. We have it, and we are thankful for it. We would miss it if it were taken away, and we feel a little pity for the so-called color blind, who see colors but seem to see them "incorrectly." But if we are asked *why* we should have color vision, we may find it difficult to give reasons other than purely esthetic ones. In those manipulations of our environment which we lump under the headings of "culture" and "civilization," we use colors largely to give ourselves pleasure. Apart from their use in labeling, in signals and uniforms, colors find very little strictly practical employment; and we could easily devise substitutes for them if we did not have them.

However, color vision is no human monopoly. Hosts of animals have it, and they certainly did not go to the trouble of evolving it merely "for fun"—though some, like the bower-birds, have secondarily come to derive true enjoyment from color. If we wish to concern ourselves with the question of why color vision was ever invented, we not only need not confine our attention to the values of color vision for humans, but we must look for the values it has for lower vertebrates. For, we must keep in mind that color vision was first developed by animals to whom magazine covers, pretty dresses, and traffic lights mean absolutely nothing.

During the latter half of the last century, when psycho-physiologists were in a welter of experimentation in an effort to decide between the major theories of color vision then being propagated, the first accurate work on animal color vision began. The underlying hope of the animal experimenters was that they would discover, among various vertebrates, various systems of color vision representing successive stages through which the human color-vision mechanism had evolved. It might then be easier to choose between theories of human color vision; and it might be possible to interpret the various human deficiency-systems (protanopia, deuteranopia, tritanopia) as throwbacks to ancestral stages of color-vision evolution.

This hope was never realized. No color-seeing vertebrate below the primates (the lemur-monkey-ape group) has ever been found to have anything but a trichromatic or three-primary system. Wherever the laws of color mixture, and the existence of complementaries, chromatic induction, closed color circles, spectral maxima of hue-discrimination, etc., have been tested in animals, these have turned out to be just about the same as in man when allowances are made for the effects of such things as the intra-ocular filters which occur in many animal eyes.

The decades of study of the distribution of color vision in vertebrates have thus yielded no play-by-play account of the course of human color-vision evolution. But the data of distribution, when considered together with the distribution of certain other things, do shed light upon the "cause" of color vision. That is, we are now able to state good reasons why color-vision systems have been desirable for organisms devoid of any esthetic sense.

In its very broadest aspect, "visual acuity" means the ability to perceive accurately the details of the pattern of visual space. The capacity for distinguishing differences in brightness has long been hailed as the most important factor in visual acuity. If two adjacent grey areas are to be perceived as discrete, and if their mutual boundary is to be evident as a line or contour, then the two areas must differ appreciably in shade or tone. An animal which lacks color vision can perceive contours only upon this basis, for to him all things are grey. But it is obvious that he will see no boundary between areas of equal subjective brightness even when those areas appear different in color to an animal that does have color vision. One may crudely say that for a color-seeing animal the maximum possible number of visible contours is represented

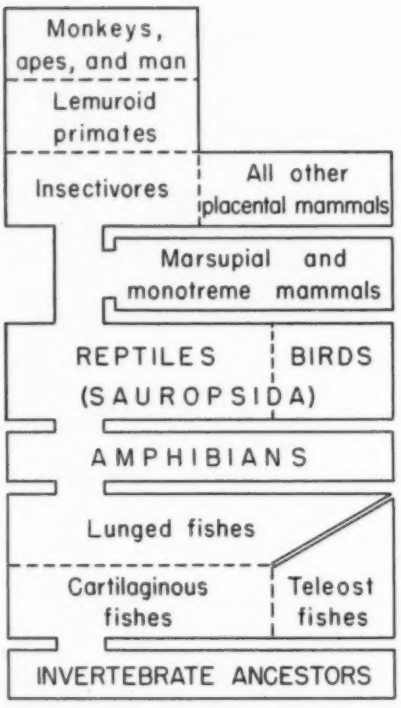

FIG. 1. Evolutionary "tree" showing the relationships of the major groups of vertebrate animals. The five Classes are the fishes, amphibians, reptiles, birds, and mammals.

by the product of his brightness-discriminating power and his hue-discriminating capacity, whereas for the totally color-blind animal the limit is set by brightness-discriminatory ability alone.

We may tentatively postulate that color vision has been evolved for the enhancement of the over-all resolution afforded by the visual system. A test of this hypothesis can take the form of a search for correlations between the presence of color vision in various animal groups and the presence of devices and phenomena which are unquestionably basic to high visual acuity.

The vertebrate groups which have been found by the best experimental methods to contain color-seeing species are the Teleostei (bony fishes), the Sauropsida (=reptiles+birds), and the placental-mammalian order Primates.* The relationships of these groups are brought out in Fig. 1, from which it can be seen that they are not directly connected. Moreover, the teleosts, birds, and primates are all terminal "blind-alley" groups from which no higher forms have been derived. Though the ancestry of avian color vision was directly reptilian, the teleosts, sauropsidans, and primates form isolated "islands" of color vision, implying that this capacity has been invented more than once in vertebrate history.

An at-once obvious common denominator of these color-perceiving groups of animals is *diurnality*. Many teleost fishes, nearly all turtles, most lizards, most snakes, most birds, all but one of the monkeys, and all of the apes are most active in the daytime. Apart from these forms, diurnal vertebrates are few and far between. Now, it is no accident that diurnality and hue-discrimination are associated, for they have a common basis in the structure of the retina.

Since Max Schultze first promulgated it in 1866, the "Duplicity Theory" has been a potent tool for the analysis of comparative ocular anatomy. It states that cone visual cells afford sharp, chromatic vision in bright light only, while rod visual cells yield unsharp, achromatic images but do so in even very dim light. There are many cone-bearing animals which are now known not to have color vision, but there are no known color-seeing animals whose retinae contain rods alone. Again, there are nocturnal animals which have cones as well as rods; but the retinae containing the most cones, and those having only cones, are all in diurnal animals.

One of the chief reasons for the association of cones with diurnality can be seen at a glance in a diagram of the neuronic hook-up of the retina (Fig. 2). Ideally, each cone has a private conduction pathway to the brain, while rods are invariably connected in multiple to optic nerve fibers. One would not expect a cone to be able singlehandedly to initiate an optic-nerve impulse in a weak illumination; but the rods, since they "gang up" on the nerve cells connected with them, are able to do so.

The greater sensitivity of the rod-mechanism of the duplex retina is, however, gained at a sacrifice of resolving power, for a point-for-point correspondence between the objective and the cerebral images is approachable only through the

* For further information on this point, and other factual data on which this paper is based, see G. L. Walls, *The Vertebrate Eye* (Cranbrook Institute of Science, Bloomfield Hills, Michigan, 1942).

medium of the cone-mechanism. As might be expected, it has been found that brightness-discrimination is better in bright light than in dim. Since the cones afford sharp vision and color sensations, and are out of action in dim light, we can see that a common basis exists in the retina for diurnality, high visual acuity, and color vision. And, it is an established fact that the sharpest vision occurs in diurnal vertebrates: in certain teleosts, in turtles and lizards, in birds, and in the higher primates.

The very sharpest vision is always associated with the presence of a *fovea centralis*. This is a highly modified spot of retina, employed for "fixation"—looking *at* things—in which about everything possible has been done to increase retinal resolving power locally (Fig. 3). Rods are eliminated from this region and the cones are slenderized, tightly packed, and connected to optic nerve fibers with a minimum of summation or even singly. There may be, as in primates, a local deposit of yellow pigment in the retinal tissue which intervenes between the visual cells and the light, to combat the chromatic aberration of the dioptric apparatus of the eye; and the dimple on the inner surface of the retina, which gives the fovea its name, acts as a magnifying device to spread the image over a greater number of visual cells.

A fovea has been found in a number of teleosts, and among reptiles in *Sphenodon*, one turtle, a couple of snakes, and nearly all diurnal lizards. In birds there is a fovea—sometimes two in each eye—in all but a few species. All apes have it, and all monkeys except the uniquely nocturnal *Aotus*. Thus, color vision is associated not only with the abundance of cones as such, but also with the development of exceptionally high visual acuity through the construction of a pure-cone retinal patch of maximal resolving power.

An eye which has a cone-rich retina, even if not a fovea, deserves a good optical image to work with. In nocturnal, cone-poor or pure-rod animals on the contrary, the final resolution of the cerebral image is inevitably so crude at best that it is quite unimportant whether the objective retinal image is refined or not. One may then logically expect that if color vision is really associated with high visual acuity, it will be found that the animals which have it also have well-developed and precise mechanisms for maintaining a sharp image on the retina despite to-and-from movements of the visual object—that is, good mechanisms of accommodation.

Of the various groups of fishes, only the teleosts have a conspicuous apparatus of accommodation. In these forms the eye is myopic when at rest, and the spherical lens can be drawn backward and temporally by a husky little muscle to focus distant objects and swing the line of sight more nearly straight ahead. In other fishes known to have any accommodation at all, weak muscular action moves the lens forward, and its prompt contact with the cornea limits the possible scope or range of accommodation.

The reptiles and birds share a superb method of accommodation, in which the periphery of the

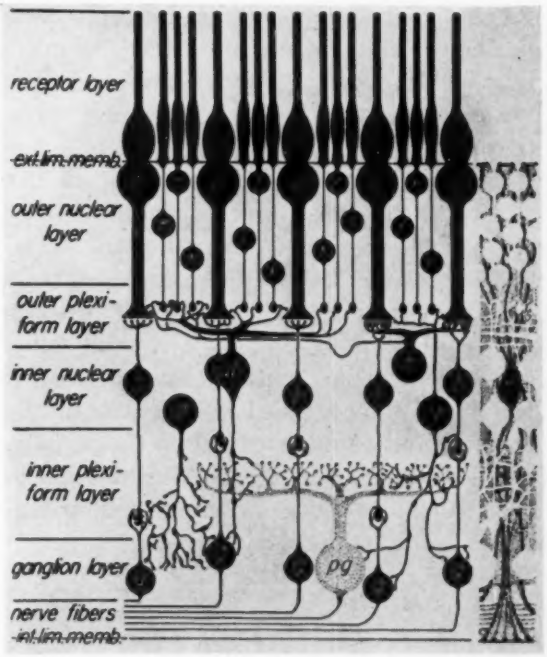

FIG. 2. Diagram of human retina, ×500, showing samples of its constituent elements. *c,c*—cone visual cells; *r,r*—rod visual cells; *b,b*—bipolar nerve cells connecting cones singly with *g,g*—ganglion cells, whose axons form the nerve-fiber layer and comprise the optic nerve; *db*—diffuse bipolar, which gathers impulses from many rods (and some cones); *pg*—parasol ganglion cell, which embraces several bipolars; *cb*—centrifugal bipolar; *h*—horizontal cell; *a*—amacrine cell; *m*—Müller fiber (supporting elements, filling all space in the retina not occupied by nerve-cell bodies and their processes).

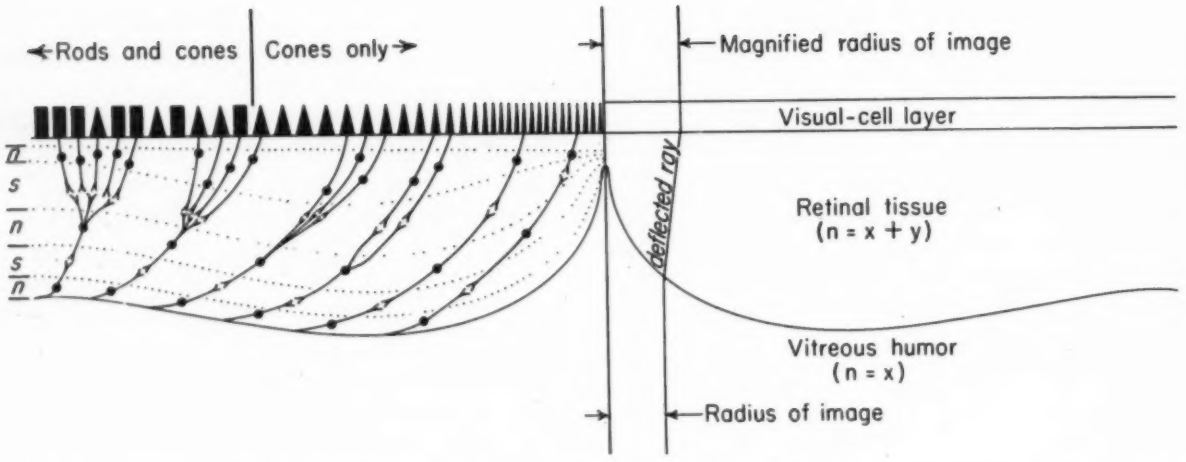

FIG. 3. Diagram of a fovea, based upon that of a lizard or bird. At the left it is shown that toward the center of the fovea the rods disappear and the cones are slenderized and closely set, while the number of visual cells hooked up to each of the nerve fibers on the inner surface of the retina is progressively reduced to one (*n,n*—layers of nuclei; *s,s*—layers of synapses; compare Fig. 2). At the right, the magnifying action of the foveal pit is depicted—the extent of this being dependent upon the value of "y," which is the difference of the refractive indices of the retinal tissue and the vitreous.

lens is directly and powerfully squeezed by a muscular ciliary body, creating a sharply curved bulge on the anterior face of the lens and greatly increasing its focusing power. Lens and ciliary body are never out of contact. They were brought into this relationship in the first place, phylogenetically, through the development of three characteristic features of the sauropsidan eyeball: a bone-supported concave zone of the sclerotic coat just behind the cornea, which has the effect of keeping the ciliary body in toward the axis of the eyeball; an array of "ciliary processes" extending radially inward from the ciliary body to meet the rim of the lens and fuse therewith; and an annular pad on the periphery of the lens, which has no dioptric function but can be thought of as an attempt on the part of the lens to meet the ciliary processes halfway.

The mammals, like the birds, had reptilian ancestry (Fig. 1). In their beginnings they were all nocturnal, and the group lived thus long enough to discard the scleral bones and lens pad as useless baggage. The standard condition of a mammalian eye is consequently one in which the sclera is everywhere convex, with the lens and ciliary processes entirely out of contact.

Among the newer mammals, wherever there has been a need to evolve a mechanism of accommodation, principles quite different from sauropsidan ones have had to be employed. The force which changes the shape of the lens is only the elasticity of the latter's capsule, and the contraction of the ciliary muscle merely releases this force by letting up, temporarily, the tension which normally exists in the suspensory-ligament fibers that anchor the lens to the ciliary body and keep it flattened when the eye is "at rest"—i.e., adjusted for distant objects. This mammalian mechanism of accommodation has been decently developed in the flesh-eaters, and perhaps also in the squirrels (which, like the higher primates, are exceptional—for mammals—in their diurnality). It is best developed by far in the monkeys, apes, and man, which are the only mammals which have a fovea—or color vision.

It may fairly be concluded that color perception has been evolved only by groups of animals which already had laid a basis for excellent visual acuity. A cone-rich retina predisposes to diurnal activity and fine brightness-discrimination, possesses high intrinsic resolving power through its low summation—and can make room for more through the creation of a fovea; and it deserves, and gets, a good image which is kept sharp by extensive and rapid accommodation. When a vertebrate has produced these things, and only then, he can get further benefit from a hue-discriminatory mechanism—provided he can evolve one.

There are indications that the evolutionary production of a color-vision system is not necessarily difficult. The electroretinogram obtained with chromatic stimulation shows characteristic differences with respect to wave-length even in animals which are known positively to have no color vision—the cat, for example. A peripheral hue-analytic mechanism, presumably photochemical in nature, may thus exist (and perhaps for some unknown reason *necessarily* exists) in any retina which contains cone visual cells. For the frosting of color vision to be added to the cake of preexisting resolving power, it is perhaps only needful for the animal to differentiate the cerebral sensory processes associated with the activities of the separate parts of the retinal wave-length-sorting equipment. And, it is perhaps unnecessary for this process to go through any intermediate "stages" at all.

Whatever does have to be done, it is only the animals to whom color vision was of life-saving value that have taken the trouble to do it. The culminant type of one of the color-seeing groups —the primates—is man, who has evolved an intelligence which can protect the abnormal, "color-blind" individual, and thus can substitute for normal color vision. Man's hue-perceiving capacity has thus been released to serve largely esthetic purposes; but this can only be interpreted as a fortunate accident.

Résumé of Recent Research

A Statistical Theory of Liquids In recent years considerable success has been obtained in the statistical treatment of liquids by the so-called cell method. This method, however, gives the liquid a quasicrystalline structure and leads into serious difficulties. A new approach to the problem has been made recently.[1] Its main features are that it restores the random distribution of the molecules (which are treated as classical centers of force) and that it includes surface tension.

The latter point proves to be of theoretical interest since it is shown that the possibility of coexistence of the liquid and vapor phase can be proved only if a transition layer is taken into account. The characteristic parameter of the theory is the "average distance of nearest neighbors." Its dependence on molecular density and temperature is determined by the combination of a well-known argument in geometrical probability with Boltzmann's theorem.

The system consisting of liquid, vapor, and surface layer is treated as a statistical unit. This leads automatically to expressions for the vapor pressure and the surface tension. The "equation of state of the surface layer" can be brought into the suggestive form

$$\gamma S^* = RT,$$

where γ is the surface tension and S^* a surface containing one mole of substance.

If the special form

$$S(r) = f/r^{\lambda} - g/r^{\mu}$$

is chosen for the potential of molecular interaction the following empirical coefficients become calculable (from 5 disposable constants): the heat of vaporization L, the coefficient of expansion a, the bulk coefficient b, the specific heat of the liquid c_p, the surface tension γ, the Eötvös constant k_E, and the Kamerlingh Onnes constant $K = RT_c/p_cV_c$.

The degree of approximation which can be realized, for ten liquids of widely different character, will best become evident from Table I. The numbers represent the ratios of theoretical to empirical values. There remain discrepancies of a systematic nature (probably due to association)

[1] G. Jaffé, Phys. Rev. **62**, 463 (1942). Part II will be published soon.

TABLE I. Comparison of theoretical and empirical data. The numbers represent ratios of theoretical to empirical values.

Liquid	L	a	b	c_p		k_E	K
Mercury	1.00	1.22	0.91	0.79	1.00	0.91	
Bromine	1.00	0.97	1.28	0.90	1.00	0.98	
Carbon disulfide	1.00	0.98	1.27	0.87	1.03	0.89	
Acetone	1.00	0.92	1.20	0.94	1.38	1.12	0.71
Carbon tetrachloride	1.00	1.01	1.27	0.79	1.14	0.98	0.76
Chloroform	1.00	0.99	1.06	0.84	1.17	0.97	
Ethyl ether	1.00	0.92	1.11	1.00	1.28	0.91	0.72
Benzene	1.00	0.96	1.19	0.89	1.20	1.02	0.76
Ethyl alcohol	1.00	1.07	0.82	1.00	1.97	2.22	0.71
Water	1.00	1.91	0.68	1.28	1.35	2.07	0.54

but the simple potential chosen coordinates the facts in a general way which must be considered satisfactory.

New Books

Dictionary of Radio and Television Terms

BY RALPH STRANGER. Pp. 252+vii, Illustrated, 15 ×22½ cm. Chemical Publishing Company, Inc., Brooklyn, New York, 1941. Price $2.50.

This is a small glossary of terms for the uninformed which promises to have very limited usefulness to anyone acquainted with ordinary physics. The author covers a rather large field, not excluding astronomy and chemistry, with definitions and information on the chemical elements, with departures into the alluring field of atomic structure (including the neutrino), and with occasional biographical notes on men whose names are associated with electricity. It is inconceivable that any intelligent "ham" would not already know most of the radio terms included in the volume. There is a strong British flavor in the vocabulary, and many of the terms commonly used in the American jargon of radio are translated into the corresponding British jargon, apparently with the primary purpose of reaching a British audience.

One can find here and there interesting bits of information, but I still find reading an ordinary English dictionary more entertaining and more informative. The book is not free from errors both in typography and in fact, and certain of the definitions leave much to be desired. For example, consider the following three taken from the H's: "HENRY.—This is the practical unit of self-inductance. A coil is said to possess a *self-inductance* of 1 henry when a current flowing in it and changing in magnitude at the rate of 1 ampere per second will induce *in another coil* an E.M.F. of 1 volt." (The italics are mine.) "HOMOGENOUS. (The spelling is his.)—A term used to describe the inner structure of a substance. A homogenous substance is a substance which is uniform—i.e. of the same kind throughout. Examples of homogenous substances are air or pure water. Granite is not a homogenous substance—it is made of different particles." "HORIZON.—This is the scientific name for skyline. Owing to the curvature of the earth, we can see only a limited distance. If our eye is at a height of h feet, then the horizon, i.e. the limit of our vision in all directions, would be at a distance of $\sqrt{\frac{3h}{2}}$." No chance for a "Lost Horizon" there, for a six-foot man would have a horizon just at arm's length! Anyone who knows that the author probably meant "the square root of three-halves *h miles*" is too erudite for this dictionary. A random selection of other definitions might not be so damning as the three just quoted, but I find a good many which are inaccurate or insufficient. An estimate of the number of terms strictly connected with television indicates that that number falls distinctly below an average of one reference per page. If your hobby is collecting dictionaries, here is one which you should have to make your collection complete; but if your interest is in radio and television and if you are more than the rankest of amateurs, you scarcely need this book.

RICHARD M. SUTTON,
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Elements of Heat Transfer and Insulation

BY MAX JACOB AND GEORGE A. HAWKINS. Pp. 169 +xvi, Figs. 50, 15½×23½ cm. John Wiley & Sons, Inc., New York, 1942. Price $2.50.

This book is intended to serve as a textbook on heat transmission for the average junior or senior engineering student. The authors have written a text covering the basic principles of heat transfer and insulation in a nonrigorous manner which will give the student useful basic information in the various phases of the subject.

As would be natural in such a text, the three methods of heat transfer—conduction, convection, and radiation—are discussed quite briefly but clearly. Under thermal conductivity, conversion factors for different units are given. The factors affecting the conductivity of gases, liquids, and solids are pointed out. The characteristics of materials useful for low temperature work, building materials, for heating and process work, for use in the power generating field, high temperature applications, as well as metals and alloys are treated. Consideration is given to the influence of temperature and moisture on the thermal conductivity of different materials. Under consideration of the steady state, the passage of heat through plain walls, composite plain walls, and cylindrical walls are treated. The idea of heat resistance is introduced as well as that of variable conductivity. Under the unsteady state, the results obtained from numerous researches are taken and illustrated by suitable sketches and diagrams, including the temperature gradient in a wall, the temperature distribution, and rate of heat flow. The special problems such as the transfer through a thick slab, a sphere, and cylinder, when subjected to a sudden surface temperature change, are discussed. The method of handling problems involving the periodic change of surface temperature is clearly stated.

Under heat conduction, brief consideration is given to the transfer of heat and temperature distribution in plane plates in which heat is produced homogeneously such as in a flat coil of wire.

The concepts and advantages of dimensional analysis as

applied to convection are briefly but clearly discussed. Attention is called to the most important of the dimensionless groups such as the Reynolds number, Grashof number, Prandtl number, and Nusselt number. Under free convection, the customary formulas are given and their use outlined for plane horizontal and vertical surfaces and pipes. The concept of a gas film is also introduced and its importance emphasized.

The standard formulas derived by Hottel and McAdams for transfer of heat to and from flowing fluids are given and applied to streamline and turbulent flow. Losses from single tubes, wires, and banks of tubes are considered with illustrative examples. The heat transfer along rods, heated at one end, and similar problems are treated from the standpoint of the combination of effects due to conduction and convection. In such cases the part due to radiation is combined with convection loss into one term. These considerations are applied to five classes of heat exchangers. The concept of *log of mean temperature difference* is introduced and applied to heat exchangers. Also the problem of the heat transfer from a wall at uniform temperature when suddenly brought into contact with a medium at different temperature is illustrated with suitable example.

Heat transfer during condensing and boiling of fluids is briefly covered and illustrated by problems.

The chapter on radiation treats this loss from the wave motion point of view. The usual subjects related to the topic are treated, including blackbody, Planck's radiation law, Kirchhoff's law of radiation and emission, Stefan-Boltzmann law, and exchange of energy between parallel planes of the same and different emissivities, and the exchange of energy between an enclosed body and the enclosure. The combined effects of conduction, convection, and radiation are discussed in connection with such problems as losses from bare and insulated steam pipes and across air spaces. The latter is treated very superficially.

The methods of determining thermal conductivities and emissivities is taken up very briefly and in fact too much so. The precautions required to determine temperatures accurately by means of thermocouples and thermometers are well pointed out indicating why errors in such measurements are so frequently made. Brief mention is made of radiation pyrometers as temperature measuring devices.

Reynolds' analogy between fluid friction and heat conduction is discussed and the equations based on Reynolds' analogy given.

The book has a well arranged set of problems after each chapter which will greatly assist the student in learning the practical use of the material covered. The authors give a very detailed list of symbols which they have used quite logically. However, the symbols for the various units are not entirely in harmony with those adopted by the American Standards Association. Some students may find this fact a little confusing. It is unfortunate that entire agreement does not exist. There is also another confusing condition in that the same symbol is used for different units such as E for emissivity and electrical potential, M for mass and mass velocity, R for thermal resistance and electrical resistance, T for absolute temperature and the fundamental unit of time, τ for time and transmissivity of radiation. The authors, however, are to be complimented on getting together so many of the useful practical relations into a concise usable form. The book is bound to be of real service to the average engineer interested in heat transfer problems.

T. Smith Taylor,
Newark College of Engineering

Here and There

Honors and Awards

A. Lewis MacClain was presented with the Octave Chanute Award at the Honors Night Dinner of the Institute of Aeronautical Sciences in New York on January 26. He is an aircraft engine test pilot and engineer of the Pratt and Whitney Aircraft Division, United Aircraft Corporation, East Hartford, Connecticut. It was given to him in recognition of his outstanding work in flight testing of aircraft engines and his development of the engine torque indicator which measures brake horsepower.

Dr. George E. Hale, the noted American astronomer who died in 1938, designer of the 100-inch telescope at the Mt. Wilson Observatory and projector of the huge 200-inch reflector now being built for the new observatory at Mt. Palomar, was commemorated by the launching of the victory ship George E. Hale, built by the California Shipbuilding Corporation.

Commander F. W. Reichelderfer, chief of the U. S. Weather Bureau since January 1939, was chosen as this year's recipient of the Robert M. Losey Award in recognition of outstanding contributions to the science of meteorology as applied to aeronautics.

Gerard Swope, President of the General Electric Company, was awarded the Hoover Medal by the American Institute of Electrical Engineers; the medal is a recognition of preeminent engineering service which has advanced the well-being of mankind.

Dr. William Wilson, who has been with the Western Electric Company and the Bell Laboratories since 1914, was awarded the Medal of Honor by the Institute of Radio Engineers. He was given the award for his achievements in the development of modern electronics, including its application to radiotelephony.

Dr. Karl T. Compton, President of the Massachusetts Institute of Technology, received the doctorate of laws from Tufts College at the first mid-winter graduation ceremonies held on January 26.

Harry E. Newell, Assistant Chief Engineer of the National Board of Fire Underwriters, was awarded the James Turner Morehead Medal, awarded annually by the International Acetylene Association for "outstanding work in the acetylene industry."

Professor G. W. Stewart, of the State University of Iowa, was presented with the Oersted Medal at the New York meeting of the American Association of Physics Teachers, which was held in conjunction with the American Physical Society. The medal is given for eminence as a physics teacher.

The following names appeared in the British New Year honors list: *Knights:* W. A. Stanier, Chief Mechanical Engineer L.M.S. Railway and scientific adviser to the Minister of Production; J. Wright, Chief Engineer, Central Electricity Board. *C.M.G.:* D. Yates, a leading metallurgist of South Australia. *C.I.E.:* R. A. MacGregor, Chief Metallurgist, Department of Supply, Calcutta.

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Staff Changes

S. Caplan, who for the past nine years has been associated as research chemist with the Harvel Research Corporation, has become the Research Manager and Acting Technical Director of the Irvington Varnish and Insulator Company, Irvington, New Jersey. Mr. Caplan succeeds C. F. Hanson who has been appointed Chief Consulting Engineer and will be responsible for expediting technical work on war production.

Theodore G. Sullivan, formerly with the Harmon Color Works, Vansul, Inc. and the Wishnick-Tumpeer Company, has joined the staff of Foster D. Snell, Inc. as Research Director of the recently expanded Rubber and Plastic Department.

John W. Buch, Assistant Professor in the Department of Mining Engineering of the Pennsylvania State College, has been named chief of the Coal Economics Division of the U. S. Bureau of Mines.

Professor A. E. Douglass, formerly director of the Steward Observatory of the University of Arizona, has been recalled from retirement. This was necessitated by the absence of Dr. Edwin F. Carpenter, who is now on the staff of the Officers School at Ithaca, New York.

Harold P. Westman, formerly secretary of the Institute of Radio Engineers, has joined the staff of the American Standards Association to spend full time on a study of War Standards for Radio.

Professor Richard Foster Flint, of the Department of Geology of Yale University, has been granted leave of absence. He has been commissioned a major in the Army Air Corps.

Dr. Ernest W. Reid, research chemist, Director of the Commodities Bureau of the War Production Board, has been appointed Director General for the Industry Division.

Dr. E. C. Williams, Chemical Director and Vice President of the General Aniline and Film Corporation, has been placed in charge of its new research laboratory at Easton, Pennsylvania. The laboratory has a staff of fifty chemists, physicists, and chemical engineers.

Professor Henry A. Boorse, of the Department of Physics of Barnard College, Columbia University, has leave of absence for the spring semester to engage in government work. His lectures will be given by Professor Robert B. Morrissey, of the Manhattanville College of the Sacred Heart.

Dean Ivan C. Crawford, of the College of Engineering of the University of Michigan, has been named Technical Adviser and Consultant to the Training Division of the U. S. Navy.

Dr. John R. Lewis, Professor of Chemistry in the University of Utah, has become Professor and Head of the Department of Metallurgy.

Dr. J. S. Swearingen, Professor of Chemical Engineering at the University of Texas, has been granted a leave of absence to join the Office of Scientific Research and Development.

Dr. Leo S. Mason, Head of the Department of Chemistry at the North Texas Agricultural College, has been appointed Professor of Chemistry at the University of Pittsburgh and Senior Research Fellow in Physical Chemistry.

Dr. L. Grant Hector, Professor of Physics at the University of Buffalo, has resigned to become Director of Engineering for the National Union Radio Corporation, of Newark, New Jersey.

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News of Societies

The American Physical Society elected officers at its meeting in New York in January. Dr. A. W. Hull, of the General Electric Company, was elected President to succeed Dr. P. W. Bridgman, Hollis Professor of Mathematics and Natural Philosophy at Harvard University. Other officers elected were Dr. Arthur J. Dempster, of the University of Chicago, Vice President; Dr. Karl K. Darrow, of the Bell Telephone Laboratories, Secretary; and Dr. George B. Pegram, of Columbia University, Treasurer.

The Mathematical Association of America has elected new officers. Professor W. D. Cairns, of Oberlin College, has been elected President; Professor C. C. MacDuffee, of Hunter College, has been elected Second Vice President; Professors Saunders Mac Lane, of Harvard University, and E. J. Moulton, of Northwestern University, Governors at Large, and Professor B. W. Jones, of Cornell University, Associate Secretary. On January 1, Professor W. B. Carver, of Cornell University, became Secretary-Treasurer. The office of the association is now in McGraw Hall, Cornell University, Ithaca, New York.

Electronics at 1943 Machine Tool Forum

The application of electronics to metal working tools will be explained by electronic engineers as one of the features of the 1943 Westinghouse Machine Tool Forum, scheduled to be held at the company's East Pittsburgh, Pennsylvania, works, April 6th and 7th. This will be the eighth annual forum sponsored by the Westinghouse Electric and Manufacturing Company. The forum is planned to provide an opportunity for machine tool engineers, representatives of machine tool user companies, and executives of machine tool manufacturers to discuss electrical problems relating to design and function of machine tools.

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Location of New and Rare Instruments

The Committee on Location of New and Rare Instruments has the following requests and offers:

Instruments Requested

Cathetometer (32″ ±0.003″)
Two-circle reflecting goniometer
Loewe Zeiss liquid interferometer
Quartz microscope
Zeiss optimeter
Hypervac vacuum pump (Cenco No. 93,000)
Refractometers: Bausch & Lomb and Zeiss-Pulfrich
Quartz spectrograph

Instruments Offered

Leeds and Northrup thermionic amplifier No. 7673
Alb. Rueprecht & Sohn analytical balances
Roller-Smith Co. surface tension balances
Bausch & Lomb centrifuge
Gaertner chronograph
Fisher electropode
Leeds & Northrup microammeter Type R
Weston microammeter for d.c.
Microscopes: Bausch & Lomb, Poller, Winkel-Zeiss, Zeiss
Sartorius-Werke brain microtome
Hartnack polariscope 400-mm tube
Cenco Hyvac pumps
pH meter model Chemie (Bergmann & Altmann)
Radio test instruments and equipment
Zeiss refractometer, dipping type
Bragg x-ray spectrometer
Keuffel & Esser spectrophotometers
Franz Schmidt & Haensch—spectroscope (student model), spectrometer, grating spectrograph, universal spectrograph, precision microcolorimeter, immersion colorimeter
Chemical model of the German typewriter "Adler"
G.E. wattmeter Type P3

Information concerning these offers and requests for rare instruments that can be sold, loaned, or leased for essential war or other research can be obtained from D. H. Killeffer, Chairman of the Committee, 60 East 42 Street, New York, New York.

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Largest Transformers Completed

The Sharon, Pennsylvania, plant of the Westinghouse Electric & Manufacturing Company recently completed the two largest transformers ever built there, each as big as a six-room house and weighing 257 tons. Twelve freight cars were required to ship the huge transformers, which will step up electric power from 27,000 to 132,000 volts for a new aluminum plant.

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New Methods Reducing Blackout Hazards

New developments plus experience gained during the past year are helping reduce the hazards of blackouts and dimouts and are making these protective measures more effective, Samuel G. Hibben stated at a meeting held recently at the Mellon Institute, Pittsburgh, Pennsylvania. Mr. Hibben, who is Director of Applied Lighting at the Westinghouse Lamp Division, Bloomfield, New Jersey, predicted a greater reduction in the accident toll during blackouts and dimouts once full advantage is taken of modern lighting and safety devices.

He said, "We have learned that it is quite possible to have a complete exterior blackout without sacrificing adequate interior illumination in home or factory. . . . One of the most interesting developments in blackout technique had been the use of phosphorescent and fluorescent materials." Recommending the use of luminous markers and signs to denote exits, doors, and factory aisles, Mr. Hibben also described its use on clothing and automobile exteriors. In addition, he added, phosphorescent coatings may be used to outline door knobs, switches, stairs, railings, paths to air raid shelters, and in many other locations.

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Necrology

Oskar Bolza, well-known mathematician who taught at the University of Chicago from its opening in 1892 to 1910, died July 5, 1942 in his native country, Germany, at the age of eighty-five.

F. W. Harbord, the English metallurgist, died on December 27, at the age of eighty-two.

Dr. Earle Raymond Hedrick, a member of the faculty of Brown University Graduate School in Advanced Instruction and Research in Mechanics, formerly Vice President of the University of California at Los Angeles, died February 3, at the age of sixty-six.

Dr. George Boris Karelitz, Professor of Mechanical Engineering at Columbia University, known for his work on lubrication, died January 19 at the age of forty-eight.

Dr. Leonard Magruder Passano, Professor emeritus of Mathematics at the Massachusetts Institute of Technology, died January 30, at the age of seventy-seven.

Dr. Howard M. Raymond, who retired ten years ago as president of the Armour Institute of Technology in Chicago, died January 24 at the age of seventy.

Contributed Original Research

The Penetrometer Method for Determining the Flow Properties of High Viscosity Fluids*

WESLEY W. PENDLETON
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(Received December 3, 1942)

A review of the literature has shown that there was no reliable method of measuring the flow properties of fluids in the range of viscosity 10^6 to 10^{12} poises where the flow is "generally viscous." The theory of the penetrometer has been developed in this paper to a point where absolute viscosities may be determined both for viscous flow (viscosity independent of rate of shear) and generally viscous flow (viscosity dependent on rate of shear). The new penetrometer method was checked (1) with the capillary viscometer for viscous flow using coal tar pitch, and (2) with the revised axially moving cylinder method for generally viscous flow using a high melting blown asphalt. For any one temperature, the flow properties of a generally viscous material are defined by the straight line relationship between log shearing stress and log rate of shear. From this relationship viscosity may be calculated at any desired rate of shear. The slope n of this line is required in the calculation of rate of shear both for the penetrometer method and for any other type of standard viscometer method. An independent means for observing n was developed for the penetrometer and for the moving cylinder method. It is shown that earlier formulas for standard instruments based on viscous flow are invalid when applied to fluids showing generally viscous flow properties. The power function law relating shearing stress and rate of shear has been found to hold for the asphalt studied over the range of rate of shear from 10^{-5} to 10 reciprocal seconds. In addition to speed of operation and precision, the penetrometer has the advantages of minimizing the elastic effects and reproducibility without extensive heat treatment and preworking. Entire shearing stress-rate of shear curves can be determined with one load and one penetration using a succession of time intervals.

I. INTRODUCTION

THE viscosity of asphalt is known to have a marked influence on its electrical as well as its mechanical properties. The range of viscosity of asphalts used as electrical insulation is roughly 10^6 to 10^{12} poises. The penetrometer has been found to work admirably in this range provided extensions are made to time and load which are limited in the standard A.S.T.M. test. An examination of this instrument indicated that it could be used not only to measure absolute viscosities of viscous fluids but could be used to study the flow properties of generally viscous fluids. In this paper the term "viscous" will apply to fluids showing viscosity independent of rate of shear while the term "generally viscous" will refer to fluids in which viscosity is dependent on the rate of shear. The former is considered a special case of the latter.

A review of the methods for measuring high viscosities is given by van Nieuwenberg.[1] The penetrometer is listed as being useful only over a small range of viscosities because of the A.S.T.M. limitations. The advantage of the use of longer times than used in the standard penetration test is brought out by Rhodes and Volkmann[2] who stated that the first five seconds of penetration should not be used in viscosity determinations of viscous fluids. They developed a formula which made use of successive penetrations at five-second intervals to determine an average viscosity from a group of readings. The first interval was not included in the average.

Saal and Koens[3] have developed an empirical formula, also applying only to viscous fluids, which converts A.S.T.M. penetrations to vis-

* This paper was presented at the annual meeting of the Society of Rheology held at New York City, October 30–31, 1942.

[1] C. J. Van Nieuwenberg, *First Report on Viscosity and Plasticity* (Academy of Sciences, Amsterdam, 1935), p. 147.
[2] E. O. Rhodes and E. W. Volkmann, J. App. Phys. **8**, 492 (1937).
[3] R. N. J. Saal and G. Koens, J. Inst. Petrol. Tech. **19**, 176 (1933).

cosity. This is reputed to be fairly accurate over a limited range of viscosity. The convenience of the method makes it desirable for rough comparisons of viscous fluids.

The first use of the power function relationship between depth of penetration and time, to determine viscosity, was made by Mack.[4] Mack stated that the product of the square of the depth of penetration and the exponent of the penetration-time equation characterizes the flow of a material. The method is applicable to both viscous and generally viscous fluids. Mack's work is the basis for the development of the proposed method.

In this paper formulas are developed for viscosity, shearing stress, and rate of shear in terms of penetration data. The development of these formulas was greatly influenced by the work done by Dr. A. Nadai. Correlations are made (1) at low viscosities (3.8×10^5 poises) with the capillary viscometer using data presented by Rhodes and Volkmann;[2] and (2) with the moving cylinder viscometer developed by Pochettino[5] at high viscosity (10^{10} poises) where asphalts show generally viscous flow. The correlation at high viscosity is a correlation of shearing stress-rate of shear diagrams.

II. THEORETICAL DEVELOPMENT

A. The Moving Cylinder Viscometer with a Viscous Fluid

The definition of coefficient of viscosity for a viscous fluid is given by

$$\eta=\tau/\dot{\gamma}, \tag{1}$$

where

η = coefficient of viscosity,
τ = shearing stress,
$\dot{\gamma}$ = rate of shear.

The rate of shear is the time rate of change of the amount of shear or

$$\dot{\gamma}=d\gamma/dt, \tag{2}$$

where

γ = amount of shear.

The amount of shear in a system between two coaxial cylinders, where the inner is loaded axially, may be found by

$$\gamma=du/dr, \tag{3}$$

where

u = axial displacement,
r = radius of any point in material.

The velocity of displacement of a point in the viscous fluid is

$$v=du/dt. \tag{4}$$

Substituting Eqs. (3) and (4) in (2) gives, for rate of shear:

$$\dot{\gamma}=dv/dr. \tag{5}$$

If the motion of the center rod is downward and this is taken as the positive direction, the sign of the shearing stress in the material will be negative. The relation for shearing stress in the material at radius r in terms of the shearing stress exerted by the rod is, obviously,

$$\tau=-\tau_a(a/r), \tag{6}$$

where

τ_a = shearing stress acting downward at the surface of the rod,
r = radius of any point in material,
a = radius of rod.

From Eqs. (1), (5), and (6), the following differential equation develops:

$$dv/dr=-(\tau_a/\eta)(a/r), \tag{7}$$

where

η = coefficient of viscosity.

By integrating Eq. (7) the following relation is obtained which is the same as developed by Hatschek:[6]

$$v_a=(a\tau_a/\eta)\ln(b/a), \tag{8}$$

where:

b = inner radius of outer cylinder,
v_a = maximum velocity at $r=a$.

B. The Moving Cylinder Viscometer with a Generally Viscous Fluid

If a generally viscous substance is used, a variable viscosity, still defined by Eq. (1), must be used in Eq. (7). A power function law between shearing stress and rate of shear is assumed to be:

$$\tau^n/\dot{\gamma}=\tau_a{}^n/\dot{\gamma}_a, \tag{9}$$

where:

n = constant,
τ_a = shearing stress at $r=a$,
$\dot{\gamma}_a$ = rate of shear at $r=a$.

[4] C. Mack, Proc. Assoc. Asphalt Paving Tech. **13**, 194 (1942).
[5] A. Pochettino, Nuovo Cimento **8**, 77 (1914).
[6] E. Hatschek, *The Viscosity of Liquids* (G. Bell and Sons, Ltd., London and D. van Nostrand Company, New York, 1928), p. 184.

From Eqs. (1), (6), and (9), the variable viscosity becomes:

$$\eta=\eta_a(r/a)^{n-1}, \tag{10}$$

where:

η_a = viscosity at $r=a$.

Substituting Eq. (10) in (7), the differential equation becomes:

$$dv/dr=-(\tau_a/\eta_a)(a/r)^n. \tag{11}$$

Here, η_a is a constant since τ_a is a constant, even when a generally viscous fluid is assumed.

The result of integration and substituting the limits makes the maximum velocity at the rod radius:

$$v_a=\frac{a\tau_a}{\eta_a(n-1)}\left[1-\left(\frac{a}{b}\right)^{n-1}\right]. \tag{12}$$

From Eqs. (1) and (12), the following expression for $\dot{\gamma}_a$ for the Pochettino moving cylinder method in a generally viscous substance is easily found:

$$\dot{\gamma}_a=\frac{v_a(n-1)}{a[1-(a/b)^{n-1}]}. \tag{13}$$

Since shearing stress is the force on the rod over the lateral area, we have

$$\tau_a=\frac{Mg}{2\pi al}, \tag{14}$$

where

M = mass on rod in grams,
g = acceleration of gravity,
l = length of rod in contact with fluid.

C. The Penetrometer with a Generally Viscous Fluid

The shearing stress τ_a was assumed constant for the moving cylinder case where the rod protruded from top and bottom of the fluid under test. Passing to the penetrometer we shall first assume that a hollow cylinder with infinitely thin wall and frictionless interior is pushed into a generally viscous substance. Corrections for the actual standard needle will be derived in Section D, Part II. The maximum shearing stress in this case is variable and l in Eq. (14) is replaced by the variable Z, or

$$\tau_{aZ}=Mg/2\pi aZ, \tag{15}$$

where

Z = depth of penetration.

From Eqs. (1), (9), and (15), the following relation is found to exist between viscosities and their respective penetrations:

$$\eta_{aZ}/Z^{n-1}=\eta_{aL}/L^{n-1}=B, \tag{16}$$

where

η_{aZ} = viscosity at penetration Z,
η_{aL} = viscosity at a specific value of $Z=L$,
B = constant.

In Eq. (12), let $v_a=dZ/dt$, $\eta_a=\eta_{aZ}=BZ^{n-1}$, and $\tau_a=\tau_{aZ}$ as given by Eq. (15). Thus

$$\frac{dZ}{dt}=\frac{Mg[1-(a/b)^{n-1}]}{2\pi BZ^n(n-1)}. \tag{17}$$

This equation is similar to the relation derived by Dr. Nadai in an unpublished Westinghouse report on the penetrometer relations.

Integrating Eq. (17) the result is:

$$\frac{Z^{n+1}}{n+1}=\frac{Mgt[1-(a/b)^{n-1}]}{2\pi B(n-1)}. \tag{18}$$

Replacing B in Eq. (18) by η_{aZ}/Z^{n-1} from (16), the following equation for viscosity is found:

$$\eta_{aZ}=\frac{Mgt(n+1)}{2\pi(n-1)Z^2}[1-(a/b)^{n-1}]. \tag{19}$$

Dividing Eq. (19) by (15), the rate of shear is found

$$\dot{\gamma}_{aZ}=\frac{(n-1)Z}{a(n+1)t}\cdot\frac{1}{[1-(a/b)^{n-1}]}. \tag{20}$$

All quantities in Eqs. (15), (19), and (20) are known or can be found except the exponent of the speed law n. The following development gives an expression for n which may be evaluated by means of experimental values of penetration and time.

By taking the logarithms of both sides of Eq. (9), the following relation is obtained:

$$n\log\tau-\log\dot{\gamma}=\log K_1. \tag{21}$$

Differentiating Eq. (21) gives:

$$n=d\log\dot{\gamma}/d\log\tau. \tag{22}$$

Let $c=1/n+1$ (see Mack[4]), then from Eq. (22)

$$c=d\log\tau/(d\log\tau+d\log\dot{\gamma}). \tag{23}$$

From Eq. (15) it is seen that

$$d\log\tau=-d\log Z \tag{24}$$

and from Eq. (20), it is also seen that

$$d\log\dot{\gamma}=d\log Z-d\log t. \tag{25}$$

Substituting Eqs. (24) and (25) in Eq. (23), the relation becomes:

$$c=d\log Z/d\log t. \tag{26}$$

Thus, the penetration varies with time according to the following relation

$$Z=K_2t^c=K_2t^{1/n+1}, \tag{27}$$

where K_2 is a constant.

The foregoing expressions for flow properties were derived by assuming a power function for the general speed law and then through a process of integration a relation between penetration and time was developed. As shown by Dr. Nadai, this method of attack could have been reversed. Equation (27), relating penetration and time, can also be established experimentally. Thus one might start with Eq. (27) and, by a process of differentiation, arrive at the general power function speed law expressed by Eq. (21).

A few similar cases of interest are known. One can, for example, compute from an assumed general speed law such as the one expressed by Eq. (21), the amount of fluid flowing through a capillary per unit of time as a function of the dimensions of the tube. Conversely, if this latter function is given or can be found from observations, one can determine the corresponding speed law. Another illustration might be the torsion of a round bar of a ductile metal such as copper. If the stress-strain curve is known for pure shear,

$$\tau=f(\gamma), \tag{28}$$

one can easily compute the torque as a function of angle of twist per inch of length θ or,

$$M=F(\theta), \tag{29}$$

where

M = torque,
θ = angle of twist per inch of length.

Equation (29) has been found to be

$$M=2\pi\int_0^a\tau r^2dr, \tag{30}$$

where

a = radius of bar.

Or, if the torque-angle relation, Eq. (29), has been observed, Eq. (28) for the stress-strain relation may be computed by a process of differentiation. This is

$$\tau_a=\frac{1}{2\pi a^3\theta^2}\frac{d}{d\theta}(M\theta^2) \tag{31}$$

with $a\theta$ equal to the corresponding rate of shear $\dot{\gamma}_a$ (see Nadai[7]).

D. The Corrections for Penetrometer Needle Point

In the foregoing analysis it was assumed that no asphalt was displaced by the penetrating cylinder. This correction for the standard needle has been shown by Rhodes and Volkmann[2] to be negligible.

The relation for shearing stress (Eq. (15)) is in terms of the area of the hypothetical cylinder assumed. In order to use the equation with the same diameter of cylinder for the standard needle (see Fig. 1) it is necessary that a correction factor be subtracted from the depth of penetration making the cylinder area equal to the area of the needle in contact with the asphalt. The

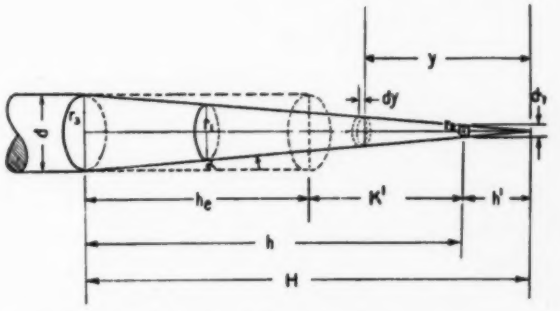

FIG. 1. Dimensions of A.S.T.M. needle:

$d=0.1$ cm,	$r_2=0.0075$ cm,
$\alpha=4.5°$,	$h'=0.095$ cm,
$H=0.635$ cm,	$h=0.54$ cm,
$d_1=0.015$ cm,	$h_e=0.31$ cm,
$r_3=0.05$ cm,	$K'=0.23$ cm.

[7] A. Nadai, *Plasticity* (McGraw-Hill Book Company, New York, 1931), pp. 127, 128.

correction factor is 23 decimillimeters for penetrations greater than 54 decimillimeters which is the altitude h of the truncated cone shown in Fig. 1.

For penetrations less than 54 decimillimeters, the correction is a variable depending on depth of penetration. The corrected length is that of an equivalent cylinder of diameter d whose lateral area is equal to that part of the truncated cone in contact with the test material.

A diagram of the standard needle is shown in Fig. 1. An analysis of the surface area relationships resulted in the following expression for the needle point correction parameter:

$$K'=Z\left(1-\frac{r_2}{r_3}\sec\alpha-\frac{Z\sec\alpha\tan\alpha}{2r_3}\right)-\frac{r_2^2}{2r_3}, \quad (32)$$

where the values of the needle constants are given in Fig. 1.

With the numerical values, Eq. (21) reduces to:

$$K'=Z(0.849-0.788Z)-0.000562 \text{ cm}. \quad (33)$$

It is conventional to state penetration in terms of decimillimeters expressed by P. Thus Eq. (33) becomes:

$$K'=P(0.849-0.00788P)-0.0562 \text{ dmm}, \quad (34)$$

where P is the penetration in decimillimeters and K' is a length correction which has to be subtracted from the penetration. Shearing stress given in Eq. (15) now becomes:

$$\tau_{aZ}=\frac{Mg100}{\pi d(P-K')}=\frac{M}{P-K'}\cdot 3.12 \times 10^5 \text{ dynes/cm}^2. \quad (35)$$

Using the constants of the A.S.T.M. cup and needle and replacing Z by P (penetration in decimillimeters), the rate of shear in Eq. (20) reduces to:

$$\dot{\gamma}_{aZ}=\frac{(n-1)P}{5(n+1)t}\cdot\frac{1}{[1-(a/b)^{n-1}]} \text{ sec.}^{-1}. \quad (36)$$

The viscosity, then, is given by:

$$\eta_{aZ}=\frac{Mt(n+1)}{P(P-K')(n-1)}\cdot[1-(a/b)^{n-1}]1.56\times 10^6 \text{ poises}. \quad (37)$$

E. Procedure for the Penetrometer Method

Since the exponent n of the power function for the general speed law can be found by a plot of log P *vs.* lot t, the method calls for at least three values of penetration and time. These data can be obtained successively with one load and without removing the needle. Values of τ_{aZ} and $\dot{\gamma}_{aZ}$ can be calculated by means of Eqs. (35) and (36) for each pair of P and t and plotted on logarithm paper. The straight line obtained may be extrapolated to a standard rate of shear. Traxler *et al.*[8] have suggested that this value of rate of shear be 10^{-3} reciprocal second for high viscosity materials. A complete definition of the flow properties of a generally viscous material requires not only a knowledge of the viscosity at a certain value of the rate of shear, but also the slope of the log τ—log $\dot{\gamma}$ curves. This information is easily obtained with the penetrometer.

III. CORRELATION OF THE PENETROMETER METHOD WITH THE CAPILLARY VISCOMETER FOR VISCOUS FLOW

From Eq. (19) it will be seen that as n approaches unity the right-hand side becomes the indeterminate form 0/0. Application of L'Hospital's rule reduces the form to:

$$\lim_{n\to 1}\frac{[1-(a/b)^{n-1}]}{n-1}=\ln\frac{b}{a}. \quad (38)$$

It was found, for low values of n, that the viscosity formula can best be checked by making the substitution indicated in Eq. (38). Thus Eq. (37) becomes, for nearly viscous fluids,

$$\eta_{aZ}=\frac{Mt(n+1)\ln(b/a)}{P(P-K')}1.56\times 10^6 \text{ poises}. \quad (39)$$

The term ln (b/a) is about 4.0 for the standard needle dimensions and thus Eq. (39) reduces to

$$\eta_{aZ}=\frac{Mt(n+1)}{P(P-K')}6.25\times 10^6 \text{ poises}. \quad (40)$$

For viscous fluids $n=1$ and Eq. (40) becomes

$$\eta_{aZ}=\frac{Mt}{P(P-K')}1.25\times 10^7 \text{ poises}. \quad (41)$$

[8] R. N. Traxler, H. E. Schweyer, and J. W. Romberg, Proc. A.S.T.M. **40**, 1182 (1940).

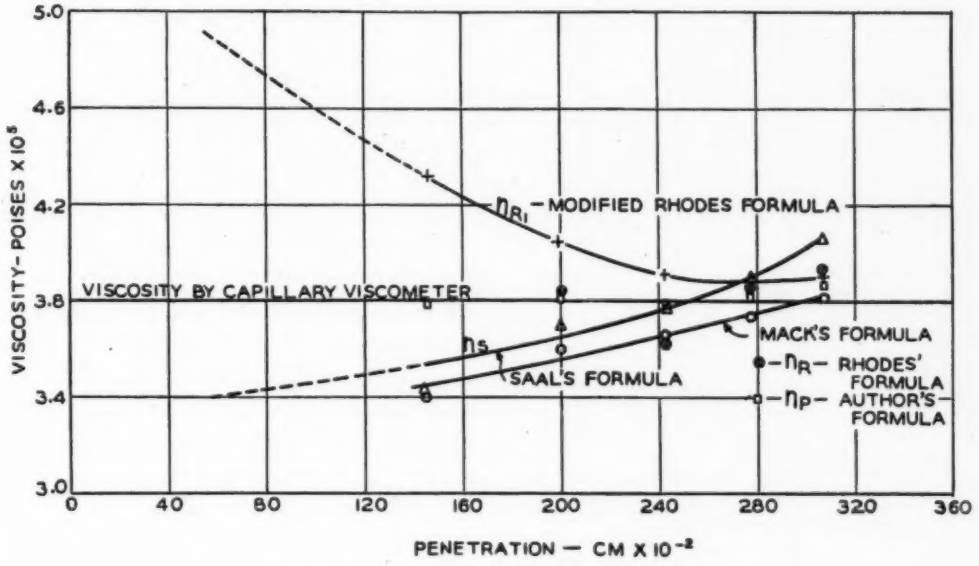

FIG. 2. Comparison of viscosity formulas using Rhodes data.

Equation (40) is similar to Mack's equation[4] which is:

$$\eta_M = ALt/cP^2, \tag{42}$$

where A = a constant, $c = 1/n+1$, and L = load.

Equation (41) is similar to the formula by Rhodes and Volkmann[2] which is:

$$\eta_R = \frac{W\Delta t \times 1.25 \times 10^7}{(P_f - P_i)(P_f + P_i - 46)}, \tag{43}$$

where

Δt = five-second time interval,
P_f = final penetration,
P_i = initial penetration.

Rhodes and Volkmann suggested that their formula, with $P_i = 0$, $W = 100$ grams, and $\Delta t = 5$ seconds, was similar to that of Saal and Koens.[3] Thus modified, the Rhodes and Volkmann formula reduces to

$$\eta_{R_1} = \frac{6.25 \times 10^9}{P(P-46)}. \tag{44}$$

The Saal and Koens formula is:

$$\eta_s = \frac{5.13 \times 10^9}{P^{1.93}}. \tag{45}$$

Using the data presented by Rhodes and Volkmann, the curves in Fig. 2 were calculated from Eqs. (40), (42), (43), (44), and (45). From a plot of penetration *vs.* time on log paper, the value of n was found to be 1.16 which indicates that the coal tar pitch was not purely viscous. It is seen that the modified Rhodes and Volkmann formula is corrected too much for the needle point while the uncorrected formulas of Saal and Mack show a reverse slope. The values by the Rhodes method using Eq. (43) are meant only for averaging and no curve was drawn through the points. However, individual points varied from the capillary line by as much as 4.5 percent. The points calculated by Eq. (40) are all within 2 percent and the general trend to higher values at higher penetrations may be due to the slight generally viscous nature of the coal tar pitch.

IV. CORRELATION OF THE PENETROMETER METHOD WITH THE POCHETTINO VISCOMETER FOR GENERALLY VISCOUS FLOW

The Pochettino[5] moving cylinder method consists of an arrangement of coaxial cylinders with the material under test cast between them. Load is applied axially to the inner cylinder. The reason for selecting this method is that it is the closest in form to the penetration test of other standard types of viscometers for measuring high viscosities.

In Fig. 3, a diagram of the apparatus is shown. The asphalt A is cast in a metal cylinder

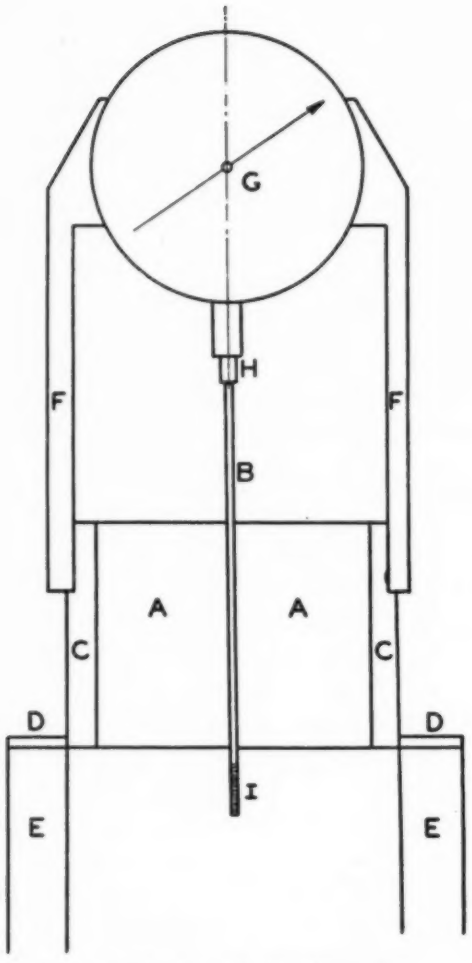

FIG. 3. Moving cylinder viscometer.

C ($2\frac{1}{2}$" inside diameter) with a metal rod B ($\frac{1}{8}$" diameter) held in place by guides (not shown) during the pouring operation. The feet D and pillars E support the cylinder C above the oven floor. The dial gauge G is rigidly fastened to C by a yoke F so that the foot H is in contact with the top of rod B throughout the test run. The rod B is loaded at I by hanging weights on an extension (not shown) through the oven floor. Eqs. (12), (13), and (14) were used for the asphalt tested which possessed generally viscous flow properties.

Some of the difficulties encountered in this experiment are given here as follows:

1. A large proportion of the total deflection was due to delayed elasticity.

2. The flow properties depended on the previous heat treatment.

3. The flow properties depended on the extent of cyclic working which breaks down the structure of the asphalt.

4. The flow properties depended on the total time of continued stress in one direction which resulted in strain hardening.

A standard heat treatment was developed which consisted in pouring the asphalt at a certain temperature into the form which was at an elevated temperature, cooling very slowly in an oven to room temperature for a definite time, and holding the temperature of the sample at test temperature (36°C) for a definite time before measurements were started. Also a standard schedule of loading and unloading to give a constant worked condition was adopted. Measurements were made at long times and at short times to determine the effect of strain hardening. The large proportion of delayed elastic deformation made it necessary to standardize on the relative amounts of flow and elastic movement. It was decided, quite arbitrarily, to run tests long enough to obtain a viscous deformation one-half as large as the delayed elastic part, or one-third of the total deflection.

The typical deflection-time curve, shown in Fig. 4, shows the viscous flow line obtained by subtracting the inverted recovery curve from the load curve. The part of the recovery curve between A and B was found to be practically an instantaneous elastic recovery which is a discontinuous function. The part between B and C was analyzed graphically in Fig. 5 by the method of

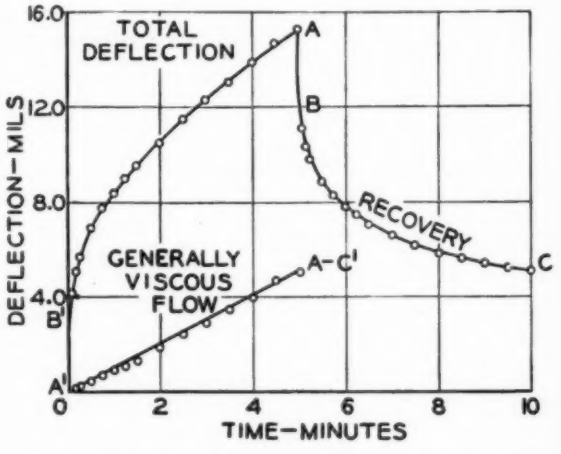

FIG. 4. Typical deflection-time curve and recovery.

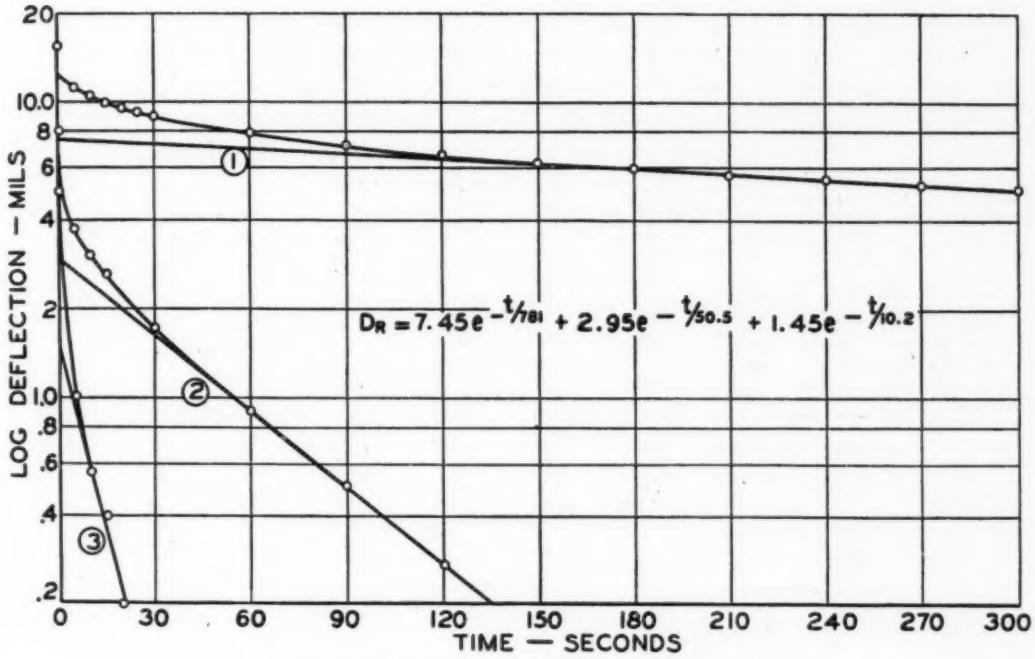

FIG. 5. Method of determining equation for recovery curve.

"successive residuals."[9] The form of the equation for the recovery curve, exclusive of the instantaneous elastic factor, is given by:

$$D_R=\frac{K}{E_1}\exp(-t/\lambda_1)+\frac{K}{E_2}\exp(-t/\lambda_2)+\frac{K}{E_s}\exp(-t/\lambda_3). \quad (46)$$

The three terms in Eq. (46) are represented by three parallel systems of spring and dashpot in series (see Fig. 6) with the moduli of elasticities of the springs given E_1, E_2, and E_3 and the time constants λ_1, λ_2, and λ_3. In the load curve the instantaneous elastic element may be introduced as a constant term. The viscous term is a function of time. Thus, for the load curve the following equation is given:

$$D_L=\frac{K}{E_0}+\frac{K_1 t}{\eta_0}+\frac{K}{E_1}[1-\exp(-t/\lambda_1)]+\frac{K}{E_2}[1-\exp(-t/\lambda_2)]+\frac{K}{E_3}[1-\exp(-t/\lambda_3)]. \quad (47)$$

[9] J. Lipka, *Graphical and Mechanical Computation* (J. Wiley and Sons, New York, 1918).

E_0 is the modulus of elasticity and is represented by a spring (see Fig. 6) in series with a dashpot of viscosity η_0. K is a constant common to all terms except the viscous term.[10]

The time constant is proportional to the ratio of viscosity to elastic modulus and, with the springs considered as perfectly elastic bars of certain dimensions, the time constants may be made equal to that ratio. For the test given in Fig. 4, $K=8.37\times10^8$ and $K_1=1.29\times10^7$. Poisson's ratio for asphalt is assumed to be 0.3.

Table I gives values of E, η, and λ for the particular asphalt tested and under the particular conditions of the test and assumptions made. The parameters vary with the stress applied.

TABLE I. Values of the model parameters.

Subscript	E dynes/cm²	η poises	λ seconds
0	2.43×10^8	7.6×10^8	
1	1.12×10^8	8.75×10^{10}	781
2	2.84×10^8	1.43×10^{10}	50.5
3	5.79×10^8	5.9×10^9	10.2

From the table it is seen that the movement of the third parallel component of the model is

[10] For a complete discussion of models see J. M. Burgers, *First Report on Viscosity and Plasticity* (Academy of Sciences, Amsterdam, 1935), pp. 21–31.

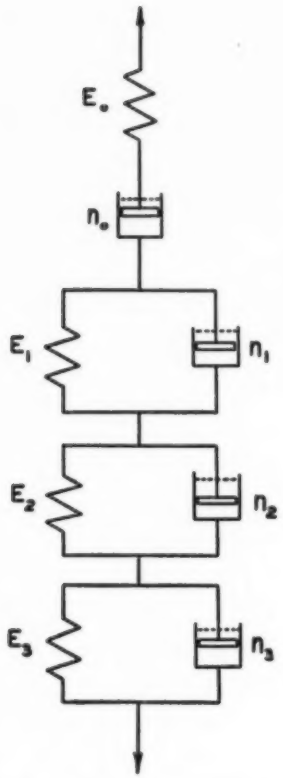

FIG. 6. Model for asphalt.

the most rapid in responding to a stress. The displacement grows to 63 percent in the first 10 seconds. On the other hand, the controlling components, at long times, are the first parallel circuit and the viscosity element η_0.

For comparison, the modulus of elasticity for steel is 2×10^{12} dynes/cm^2 or 10,000 times that for asphalt. No rigid theoretical significance should be attached to the model given in Fig. 6 for it was obtained from a graphical analysis of the data.

The discovery that a large proportion of the total movement in the falling cylinder method was due to elastic effects made it appear necessary to evaluate these effects in the penetrometer method. The elastic recovery was found by experiment to be about two decimillimeters in 150. This should be entirely negligible. The following mathematical comparison of the elastic and viscous forces acting on the penetrometer needle in the vertical direction was made to verify the experimental result. The case considered is for penetrations entirely on the conical part of the needle and it is assumed that the cone comes to a point. Distances are taken from the hypothetical point (see Fig. 1).

The vertical elastic force acting on an element of surface of the cone is given by Lethersich[11] as follows: (see Fig. 1)

$$dF_e=2\pi y dy(Z-y)E\tan^2\alpha, \tag{48}$$

where

E = modulus of elasticity,
Z = depth of penetration,
α = taper angle,
y = distance from needle point to element of surface.

When integrated between the limits $y=0$ and $y=Z$, this becomes:

$$F_e=2\pi\tan^2\alpha\cdot E(Z^3/6). \tag{49}$$

The axial viscous force on an element of surface of the cone is:[11]

$$dF_v=2\pi y dy\eta(dZ/dt)\tan\alpha(3\tan\alpha+1). \tag{50}$$

When integrated between the limits $y=0$ and $y=Z$, this becomes:

$$F_v=2\pi\tan\alpha(3\tan\alpha+1)\eta(dZ/dt)(Z^2/2). \tag{51}$$

Thus, the ratio of the viscous to the elastic force is:

$$\frac{F_v}{F_e}=\frac{3\tan\alpha+1}{\tan\alpha}\cdot\frac{3\eta(dZ/dt)}{EZ}. \tag{52}$$

Since $\eta=\tau/\dot\gamma$ and $E=\tau/\gamma$,

$$\eta/E=\gamma/\dot\gamma. \tag{53}$$

From an integration of Eq. (20) the amount of shear is given by:

$$\gamma=\frac{(n-1)Z}{a(1-(a/b)^{n-1})}. \tag{54}$$

Dividing Eq. (53) by (20) gives

$$\eta/E=\gamma/\dot\gamma=(n+1)t. \tag{55}$$

A differentiation of Eq. (27) gives an equation which when divided by Eq. (27) reduces to:

$$\frac{dZ}{dt}\Big/Z=\frac{1}{(n+1)t}. \tag{56}$$

[11] W. Lethersich, J. Soc. Chem. Ind. **61**, 101 (1942).

Substituting (55) and (56) in (52) and evaluating tan α (α for standard needle = 4.5°), gives

$$\frac{F_v}{F_e} \cong 50. \tag{57}$$

Thus the elastic force is about two percent of the viscous force acting on the conical part of the needle. With penetrations several times the length of the conical point, the elastic force would be an even smaller percentage of the viscous force. Thus, the result found by experiment, is checked.

This means that the penetrometer, at least for the asphalt tested, is essentially free from the elastic effects encountered with the moving cylinder method. This is a decided advantage for the penetrometer.

Figure 7 is a plot of log τ *vs.* log $\dot{\gamma}$ for the two methods under various conditions of test. The asphalt used had a penetration (A.S.T.M.) of 10 and a ring and ball softening point of 120°C. The value of n for this asphalt is about 3 as determined by the $\log P - \log t$ plot for the penetrometer method and the $\log v_a - \log \tau_a$ plot for the moving cylinder method. The temperature was held at 36°C±0.2°C. The lines *A*, *B*, *C*, and *D* were plotted from penetration data for loads of 100, 600, 1350, and 2350 grams, respectively. The shift to the right of the lines with increasing loads is attributed to an actual change in the material brought about by increasing amounts of working. At light loads, extremely slow penetrations are observed, giving time for the micelles to align for greatest resistance to shear. This is similar to "strain hardening" in metals. At heavy loads, penetrations are very rapid and a worked condition along the interface of the needle and asphalt results. This worked condition may be considered as a breakdown of the asphalt structure by various degrees according to the speed of the needle. Intermediate loads probably produce an equilibrium condition between these two mechanisms.

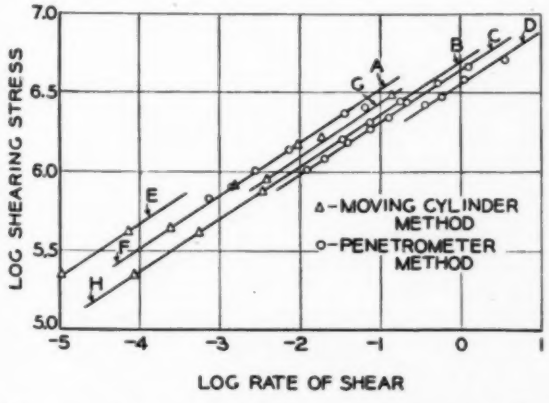

FIG. 7. Correlation of shearing stress-rate curves for the penetrometer and moving cylinder methods.

The same type of phenomenon is seen in the lines *E*, *F*, *G*, and *H* which are for the moving cylinder method. Loads ranging from 1000 to 5000 grams were used on the $\frac{1}{8}$" diameter rod. Line *E* was plotted from data taken for long time tests where the rod was loaded for 24 hours and allowed to recover for 24 hours without load. In this case the load curve reached a straight line which was parallel to the viscous flow line similar to that of Fig. 4. This represents the ultimate in unworked conditions. On the other hand, line *H* was drawn through test points following the procedure used in Fig. 4. The load was on for 5 minutes and off for 35 minutes and the process repeated until a constant viscosity was calculated. This is a much higher worked condition and the line has been shifted to the right in Fig. 7. The lines *F* and *G* are for intermediate amounts of working. It is considered fortuitous that line *A* should fall on line *F* and *B* on *H* since no quantitative correlation between amounts of working has been made for the two methods. However, this fact lends strength to the validity of the assumed speed law (Eq. (9)) as well as to the correlation between the two methods of measuring these flow properties.

V. CONCLUSIONS

Formulas developed for determining the absolute values of the flow properties of a material from penetration data have been checked by experiment for viscous and for generally viscous fluids by the use of other accepted methods of measurement.

It has been shown that the penetrometer method greatly minimizes the elastic effects present in most standard methods for measuring high viscosity asphalts. The conditions of test are less important in that no special slow cooling

procedure is necessary. The time saved by the use of the penetrometer is very great. This is brought out by the realization that each point on the curves in Fig. 7 obtained by the moving cylinder method required an average time of about one day while one complete set of points were taken with penetrometer in about 15 minutes.

The amount of working has been found to be the chief disturbing factor in determining the flow properties of an asphalt of high consistency. The formulas developed take no account of this effect and it has not been thoroughly studied experimentally in this investigation.

VI. ACKNOWLEDGMENT

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Transient Analysis of Linear Systems

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A method is suggested for determining either the indicial admittance of a linear system, when the response to a known transient is observed, or the shape of the transient, when the admittance is known. A method is also outlined for obtaining the impedance operators, and in particular the impedance coefficients (stiffness, resistance, and inertia). Corrections due to the reaction of the instrument used in measuring the response of the system, or of a restraint to insure stability of the system, can be taken into account when appreciable. Suggested applications are to acoustical and aerodynamical problems.

I. INTRODUCTION

THE indicial admittance of a linear system, or its response to a Heaviside step, or unit, function, determines completely its response to any function, transient, or steady state. This fact can be used to investigate electrical systems by impressing a unit voltage across them, and methods[1] have been recently developed for determining numerically the steady state phase and frequency characteristics directly from the indicial admittance as observed on an oscilloscope. The response *versus* frequency curve is the more commonly accepted criterion of performance of an electrical system, although many properties of a circuit can be read directly from its square wave response.[2–6] The methods of square wave analysis have not been generally applied to mechanical systems owing to the difficulty of producing a mechanical square wave. However, were the indicial admittance known, the methods of square wave analysis would be applicable, with the use of the proper electromechanical analogies. In Section II a method is given for determining the indicial admittance from the observed response to some arbitrary transient force which may or may not approximate a Heaviside function. The same method applies for determining the shape of the transient, if the admittance is known.

The impedance operators are those operators in $p(=d/dt)$ which operate on the dependent variables in the equations of motion of the system. The impedance operators are, for systems of one independent variable (time), polynomials in p and $1/p$. In the electrical case the coefficients $(L, 1/C, R)$ in these polynomials are known, or easily measurable. However, in certain mechan-

[1] Bedford and Fredenhall, Proc. I.R.E. **30**, 440 (1942).
[2] Swift, Communications **19**, 2 (Feb. 1939).
[3] Stocker, Proc. I.R.E. **25**, 1027 (1937).
[4] Bedford, Proc. I.R.E. **27**, 277 (1939).
[5] Arguimbau, Gen. Radio Exp. **14**, No. 7 (1939).
[6] Arguimbau, Gen. Radio Exp. **14**, No. 11 (1940).

ical problems, the coefficients are not known, and their values are important and intrinsic design data. Problems in stability, where these coefficients are derivatives of other force functions, fall in this category. Thus in the stability of aircraft, these coefficients are the so-called resistance derivatives, and are determinative of the stability or instability of a particular mode of flight.[7]

To proceed from the admittance function to the impedance coefficients requires only an integration. In cases where there is instability the integrals may not converge. In these cases one can apply a known restraint (e.g., for an aeroplane model, the aerodynamic balance or other support) which produces stability and convergence, and then subsequently correct for this restraint. Exactly the same procedure applies in correcting for the reaction of the measuring instrument on the system measured. The method for determining the impedance coefficients, and for applying the correction will be given in Section III.

II. DETERMINATION OF THE INDICIAL ADMITTANCE

The method rests on the fact that the performance of a system is uniquely determined not only by its response to a Heaviside unit function, but also by its response to a transient force of any form. The two are related by the theorem of Duhamel,

$$i(t)=e(0)A(t)+\int_0^t A(t-\lambda)e'(\lambda)d\lambda, \qquad (1)$$

where $i(t)$ is the response due to the arbitrary impulse $e(t)$, and $A(t)$ is the response to the unit function. $A(t)$ may be the response, $A_{ii}(t)$ in the "ith" coordinate (or circuit, or degree of freedom) due to an impulse in the "ith," or it may be the response, $A_{ij}(t)$ in the "jth" coordinate to an impulse in the "ith," with $e(t)$ bearing the first subscript and $i(t)$ the second subscript of the $A(t)$ in every case.

In the normal application one knows $e(t)$ and $A(t)$, and uses the formula to determine $i(t)$. However, if one knows or can observe $e(t)$ and $i(t)$, then the above formula, considered as an integral equation, can be used to determine $A(t)$, the indicial admittance.

Suppose that $i(t)$ and $e(t)$ are directly observed. One can solve for $A(t)$ in the following way. Break up the impressed transient $e(t)$ into elements as indicated in Fig. 1. The integral equation becomes a set of linear algebraic equations in the unknown A's. The subscript n indicates the values of the various functions at the end of the nth interval; m is the summation index corresponding to the variable of integration λ. If $e(0)=\Delta e_0$, Eq. (1) becomes

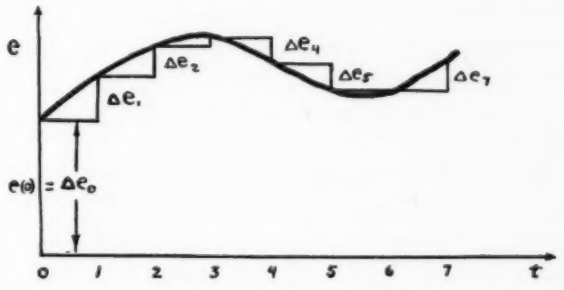

FIG. 1. Approximation of transient by elements.

$$i_n=\Delta e_0 A_n+\sum_{m=1}^{m=n} A_{n-m}\Delta e_m \quad n=0, 1, 2, \cdots. \qquad (2)$$

Written individually,

$$\begin{aligned}
i_0&=A_0\Delta e_0,\\
i_1&=A_1\Delta e_0+A_0\Delta e_1,\\
i_2&=A_2\Delta e_0+A_1\Delta e_1+A_0\Delta e_2,\\
i_3&=A_3\Delta e_0+A_2\Delta e_1+A_1\Delta e_2+A_0\Delta e_3,\\
&\cdots\cdots\cdots\cdots\cdots\\
i_n&=A_n\Delta e_0+A_{n-1}\Delta e_1+A_{n-2}\Delta e_2+\cdots\\
&\qquad\qquad +A_1\Delta e_{n-1}+A_0\Delta e_n,\\
&\cdots\cdots\cdots\cdots\cdots
\end{aligned} \qquad (3)$$

This set of equations is to be solved for the A's, given the i's and the Δe's. A similar solution will, of course, apply for determining the Δe's, given the i's and the A's.

Since the first equation involves only one unknown, A_0, and each succeeding equation introduces one more unknown, the entire set, or as much of the first part of the indicial admittance as is desired, can be directly obtained by iterated

[7] Durand, *Aerodynamic Theory* (J. Springer, Berlin, 1935), Vol. V, Sect. N.

substitution. The general term is

$$A_n = i_n - \sum_{m=1}^{m=n} A_{n-m}\Delta e_m \Big/ \Delta e_0. \qquad (4)$$

This form is suitable for machine computation, each equation requiring only the evaluation of sums of products, a subtraction, and a division. In practice, in order to facilitate the formation of the sums of products, the Δe's are listed in increasing sequence *down* one column, while the A's, as computed, are tabulated in increasing sequence *up* a column on a separate sheet of paper.

It should be noted that it is unimportant whether the i's and the Δe's, assumed observed, have their zero point in indexing adjusted precisely. Thus if, for i_0, one had zero; for i_1, i_0; for i_2, i_1, etc., then exactly the same sequence of A's would be obtained, with each subscript diminished by one. The sequence of i's could be preceded by any number of zero values, and the sequence of A's would still be numerically the same. This means simply that a constant delay, or transit time has been introduced. The shape of the response curve is unaltered. Moreover, if only the initial part of $e(t)$ is available, or is accurate, only the initial part of $A(t)$ can be determined.

The function $i(t)$ can be determined by observing the response with whatever measuring instrument is used for that purpose in the system under investigation. For example, in the case of a microphone, the indicial admittance of which is being determined, the voltage developed (proportional to $i(t)$ through a resistance) can be amplified and observed on an oscilloscope. In the general case the admittance will be a displacement (cf. charge) or its first or higher derivative, per unit force, depending on how the equations of the systems are set up. In practice it will often be simplest to convert the direct response (velocity or displacement) to a proportional voltage by an appropriate electro-mechanical coupling, amplify, and record on a cathode-ray oscillograph.

The Δe's may be determined in either of two ways. Either a device is used, the response of which is proportional to $e(t)$ itself (flat frequency response), or else a device is used, the response of which is proportional to de/dt (response increasing linearly with frequency). In the latter case $\Delta e = (de/dt)\Delta t$. The constant of proportionality is usually unimportant since in most cases it is the relative shape only of the indicial admittance which is of interest. It can, of course, be determined by calibration. Either device will have a limited range of frequency within which its response is either flat, or increases linearly with frequency, and gives a measurable signal. This range must be great enough to record the transient with at least the same accuracy as is desired for the indicial admittance.

Which of the two types of measuring device is the more suitable depends upon the following considerations:

(a) The type of transient impulse which is being used, i.e., one with large derivatives, (much high frequency composition) or one of slowly changing form (mostly low frequency composition). The best type of impulse is one which approximates a Heaviside function, since then the solution of the integral equation essentially involves applying small corrections to the observed response.

(b) The portion of the indicial admittance which it is desired to determine, i.e. $A(t)$ for t small (high frequency response) or $A(t)$ for t large (low frequency response).

(c) The range of amplitude which the measuring device can record for a given observation and the range of amplification available.

These considerations can be illustrated by an acoustical example. Consider the problem of determining the indicial admittance of a microphone to a Heaviside pressure pulse. The instrument for measuring the transient pressure impulse can be a small piezoelectric element, which, from zero up to nearly its first resonant frequency ($f_1 = \omega_1/2\pi$), has a flat open circuit response. If this piezoelectric element works into the finite resistance of a cathode-ray oscillograph amplifier, it will be flat only from ω_0 to ω_1, where $\omega_0 \backsimeq 1/RC$, RC being the time constant of the input circuit. If the range ω_0 to ω_1, is insufficient to record the transient, that is if its Fourier transform is appreciably different from zero outside this range, the range may be increased by shunting the input with a capacitance, thus extending the flat region. Alternatively the input

may be shunted with a resistance, under which condition the oscillogram will be proportional to the derivative of the impressed transient pressure. In the latter case, the lower limit is set by the noise level, the upper limit by the resonant frequency, f_1, or by $f \backsimeq 1/(2\pi RC)$, whichever is less. Either of these procedures means a loss in sensitivity, to be made up by increased amplification, or a stronger or different type of transient pressure pulse.

Reference 1 gives a method for determining the indicial admittance, when the response-frequency and the phase-frequency curves are known. The method there given is also applicable to the present case, since, by comparison with the flat piezoelectric element, the response and phase as a function of frequency can be determined, and from them the indicial admittance. The present method therefore provides an independent determination.

Since observation of single transients is often inconvenient, it might sometimes be more suitable to observe the response to a repeated transient. In order for the method outlined here to apply, the transients must be repeated at intervals sufficiently long for $i(t)$ and $e(t)$ to die away completely. For single sweep cathode-ray oscillographic recording, faster sweep speeds can be used than a speed sufficient to cross the screen in this interval, but then only the initial part of $e(t)$ and of the admittance $A(t)$ can be determined.

III. DETERMINATION OF THE IMPEDANCE COEFFICIENTS

In the general case the admittance thus determined will not be the admittance of the system alone, but of the system plus measuring instruments. In the acoustical example mentioned, the interaction is primarily that associated with the input impedance of the amplifier, as mentioned, unless very high or very low frequencies are considered. If the amplifier be flat only from ω_0 to ω_1, then this will affect the admittance $A(t)$ for $t \lesssim 1/\omega_1$ and for $t \gtrsim 1/\omega_0$, i.e., for very small and very large values of the time.

In the general case, the reaction of the measuring instruments, if not negligible, or of a restraint, can be allowed for in the following way. Let the equations of motion of the system alone be:

$$\begin{aligned} X_{11}(p)x_1+X_{12}(p)x_2+X_{13}(p)x_3&=1(t),\\ X_{21}(p)x_1+X_{22}(p)x_2+X_{23}(p)x_3&=0,\\ X_{31}(p)x_1+X_{32}(p)x_2+X_{33}(p)x_3&=0, \end{aligned} \tag{5}$$

for a unit force applied in the first coordinate at $t=0$.

$$X_{11}(p)=a_{11}p+b_{11}, \quad X_{12}(p)=a_{12}p+b_{12}, \quad \text{etc.}$$

and

$$p=d/dt.$$

For simplicity only three degrees of freedom, and first derivatives are considered. The solution for a more general case proceeds in exactly the same manner.

Now, suppose, as a result of coupling the measuring instrument or a restraint to the system, the impedance operators become,

$$\begin{aligned} Z_{11}(p)&=(a_{11}+l_{11})p+(b_{11}+m_{11}),\\ Z_{12}(p)&=(a_{12}+l_{12})p+(b_{12}+m_{12}), \quad \text{etc.}, \end{aligned}$$

where the l's and m's are known, being properties of the measuring instrument, or restraint. The equations of motion of the combination are:

$$\begin{aligned} Z_{11}(p)x_1+Z_{12}(p)x_2+Z_{13}(p)x_3&=1(t),\\ Z_{21}(p)x_1+Z_{22}(p)x_2+Z_{23}(p)x_3&=0,\\ Z_{31}(p)x_1+Z_{32}(p)x_2+Z_{33}(p)x_3&=0. \end{aligned} \tag{6}$$

This form presupposes that the addition of the measuring instrument, or restraint, does not increase the number of degrees of freedom. If it does, then the Eqs. (5) may be written with an increased number of variables, where it is understood that a_{ik}, b_{ik} are zero for k greater than the number of degrees of freedom of the system alone.

If $\Delta(p)$ is the determinant of the Z_{ij}'s and $\Delta_{ij}(p)$ is the cofactor of $\Delta(p)$ with respect to $Z_{ij}(p)$, then the solutions[8] of (5) are

$$\begin{aligned} x_1&=\frac{\Delta_{11}(p)}{\Delta(p)}\cdot 1(t)=A_{11}(t),\\ x_2&=\frac{\Delta_{12}(p)}{\Delta(p)}\cdot 1(t)=A_{12}(t),\\ x_3&=\frac{\Delta_{13}(p)}{\Delta(p)}\cdot 1(t)=A_{13}(t), \end{aligned} \tag{7}$$

[8] Carslaw and Jaeger, *Operational Methods in Applied Mathematics* (Clarendon Press, Oxford, 1941), p. 83.

where the subscripts of the three admittances indicate that they are the responses in the first, second, and third coordinate due to a unit impulse in the first coordinate.

If these three admittances have been determined, say by the methods of Section I, then the functions $\Delta_{11}(p)/\Delta(p)$, $\Delta_{12}(p)/\Delta(p)$, $\Delta_{13}(p)/\Delta(p)$, are related to $A_{11}(t)$, $A_{12}(t)$, $A_{13}(t)$ by Carson's equation.

$$\Delta_{11}(p)/\Delta(p)=p\int_0^\infty e^{-pt}A_{11}(t)=\Omega_{11}(p),$$

$$\Delta_{12}(p)/\Delta(p)=p\int_0^\infty e^{-pt}A_{12}(t)=\Omega_{12}(p), \quad (8)$$

$$\Delta_{13}(p)/\Delta(p)=p\int_0^\infty e^{-pt}A_{13}(t)=\Omega_{13}(p).$$

Since the A's are determined as graphical or tabular functions, and since the integration can be carried out by straightforward summation, the Ω's are also given as graphical or tabular functions. Actually the Ω's are needed for only a limited number of values of p. The values for $p=0$ cannot be determined directly if the areas of the A's are not finite, but it is not necessary to know the Ω's for $p=0$. If asymptotic forms of $A(t)$ for t large are available, they are useful in carrying out the integrations.

In order to determine the impedance coefficients, we have,

$$Z_{11}\Omega_{11}+Z_{12}\Omega_{12}+Z_{13}\Omega_{13}=1,$$

or

$$[(a_{11}+l_{11})p+(b_{11}+m_{11})]\Omega_{11}(p)$$
$$+[(a_{12}+l_{12})p+(b_{12}+m_{12})]\Omega_{12}(p)$$
$$+[(a_{13}+l_{13})p+(b_{13}+m_{13})]\Omega_{13}(p)=1. \quad (9)$$

This equation can be written for six or more different values of the p's, thus giving six, or more, equations in the impedance coefficients $(a_{11}+l_{11})$, $(b_{11}+m_{11})$, $(a_{12}+l_{12})$, etc. These coefficients can therefore be determined, either directly or by least squares. Judicious choice of the p's at which the value of the Ω's are taken will somewhat simplify this process. Since the l's and m's are known, the impedance coefficients a_{11}, b_{11}, a_{12}, b_{12}, etc., of the system under investigation can be determined.

To determine $a_{21}\cdots b_{33}$, requires the determination of the admittances $A_{21}(t)\cdots A_{33}(t)$, which are the responses in the various coordinates to a Heaviside pulse in the second and third coordinates.

It should be remarked that the absolute determination of the coefficients requires that the admittances be known in absolute, not relative value. If relative values only of the admittances are available, then relative values only of the impedance coefficients of the combination can be determined. To obtain relative values of the system alone, it is necessary that the impedance coefficients of the measuring instrument or restraint be determined with the same, though perhaps unknown factor of proportionality.

New Formulas for the Inductance and Reactance of Square Tubular Conductors

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Formulas are derived for the inductance per unit length of square tubular conductors. A numerical example illustrates use of these formulas. A comparison is effected with other formulas advanced earlier by Dwight and Wang.

IN two earlier papers[1,2] the writer derived alternative, numerically independent formulas that afford calculation of the inductance—and thence the reactance—of a type of bus commonly, and increasingly, used for heavy current duty: i.e., that comprised of a group of long linear conductors having parallel coplanar axes and identical, parallel-sided, uniformly thick hollow rectangular cross sections. Illustratively, in each of the papers use of the formulas derived therein is exemplified through solution of the same typical problem: calculation of the inductance of a bus comprised of square tubular conductors of specified dimensions and spacing.

Subsequent pursuit of certain considerations stemming from performance of the computation incident to these illustrative examples has resulted in new formulas that afford the inductance of *square* tubular conductors with but a tithe of the computation required by the more inclusive formulas for the general rectangular cross section. Inasmuch as most of the non-divided rectangular tubular conductor used is of square cross section, these new formulas are of evident interest to the designer of heavy duty busses. They are derived in Section 1; illustrations of their use are presented in Section 2.

1. DERIVATION OF THE FORMULAS

On the premise that the conductors are non-magnetic, right-cornered, of such length that end effects are negligible, and that the current density is uniform over the cross sections of the individual conductors—the necessity and validity of this premise are discussed in detail elsewhere[3]—the writer found the inductance of a single-phase circuit with conductor cross sections disposed as in Fig. 1 to be:

$$L=2W/w^2(rs-RS)^2 \tag{1}$$

abhenries per centimeter of bus length; wherein W, the energy of the associated magnetic field, is given by

$$W/w^2=W(r,s)+W(R,S)+2W(R+t,t)+2W(t,S+t)-2W(R+t,S+t)-2W(t,t). \tag{2}$$

Typically, $W(r,s)$ is defined by

$$\begin{aligned}W(r,s)=2r^2s^2\{&7/12+[(1/2)-(D^2/2s^2)-(r^2/12s^2)]\log[(D^2+s^2)/D^2]+[(s^2/12r^2)+(r^2/12s^2)\\&-(1/2)]\log[(r^2+s^2)/s^2]+\log(D/s)+(r^2/12s^2)\log(s^2/r^2)+(2D/s)\tan^{-1}(s/D)\\&-(2r/3s)\tan^{-1}(s/r)-(2s/3r)\tan^{-1}(r/s)-(r^4/180s^2D^2)[1-D^2(D^2-s^2)(D^2+s^2)^{-2}]\\&-(r^6/1680s^2D^4)[1-D^4(D^4-6s^2D^2+s^4)(D^2+s^2)^{-4}]\\&-(r^8/7560s^2D^6)[1-D^6(D^6-15D^4s^2+15D^2s^4-s^6)(D^2+s^2)^{-6}]-\cdots\}.\end{aligned} \tag{3}$$

All units are in the absolute system: the current density w in abamperes per square centimeter; D, R, r, and s in centimeters; W in ergs per centimeter of bus length.

Postulating that $r<s<D$; replacing the logarithmic, arctangent, and binomial terms in (3) with equivalent power series; expanding appropriately; collecting terms; and discarding those within the

[1] T. J. Higgins, Trans. A.I.E.E. **60**, 1046 (1941).
[2] T. J. Higgins, J. App. Phys. **13**, 712 (1942).
[3] T. J. Higgins, Trans. A.I.E.E. **62**, 53–57 (1943).

brackets of degree greater than eight, we have

$$W(r,s)+W(s,r)=2r^2s^2[3+2\log(D/s)+(r^2/3s^2)\log(s/r)-2\pi r/3s+25r^2/36s^2+r^4/90s^4$$
$$-r^6/840s^6+r^8/3780s^8-(2r^4-5r^2s^2+2s^4)/60D^4$$
$$-(2r^8-15r^6s^2+28r^4s^4-15r^2s^6+2s^8)/360D^8]. \quad (4)$$

Similarly, if $r=s$ and $r<D$, we obtain from (3)

$$W(r,r)=2r^4[25/12-\pi/3-(1/3)\log 2+\log(D/r)+r^4/120D^4-r^8/360D^8]. \quad (5)$$

Corroboratively, for the power series employed in obtaining (4) are yet valid for $r=s$ and $r<D$, substituting $r=s$ in (4) yields [in summing the numerical series stemming from the powers of (r/s) the discarded terms must be recalled] the expected result, twice the right-hand member of (5).

If the cross section is square ($r=s$ and $R=S$), (2) becomes

$$W/w^2=W(M+t,M+t)+W(M-t,M-t)+2W(M,t)+2W(t,M)-2W(M,M)-2W(t,t); \quad (6)$$

wherein $M=r-t=R+t$.

By use of (5) we obtain

$$W(M+t,M+t)+W(M-t,M-t)-2W(M,M)-2W(t,t)$$
$$=M^2t^2\{19.322081+7(2M^4+5M^2t^2+2t^4)/15D^4-11(2M^8+15M^6t^2+28M^4t^4+15M^2t^6+2t^8)/30D^8\}$$
$$+2\{(M+t)^4\log[D/(M+t)]+(M-t)^4\log[D/(M-t)]-2M^4\log(D/M)-2t^4\log(D/t)\}. \quad (7)$$

The logarithmic terms in the right-hand member of (7) can be transformed in such fashion that substitution of equivalent power series yields a form more convenient to numerical computation. Thus

$$-2\{M^4\log[(M^2-t^2)/M^2]+t^4\log[(M^2-t^2)/M^2]+6M^2t^2\log[(M^2-t^2)/M^2]$$
$$+4Mt(M^2+t^2)\log[(M+t)/(M-t)]+6M^2t^2\log(M^2/D^2)+t^4\log(M^2/t^2)$$
$$=M^2t^2[-14+24\log(D/M)-(4t^2/M^2)\log(M/t)-25t^2/3M^2$$
$$+2t^4/15M^4+t^6/70M^6+t^8/315M^8+\cdots\}. \quad (8)$$

Substituting (8) in (7) and evaluating (6) by use of (4) and (7),

$$W/w^2=M^2t^2[17.322081+32\log(D/M)-(8t^2/3M^2)\log(M/t)-8\pi t/3M-50t^2/9M^2+8t^4/45M^4$$
$$+t^6/105M^6+4t^8/945M^8+4M^4/5D^4+8M^2t^2/3D^4+4t^4/5D^4-34M^8/45D^8-16M^6t^2/3D^8$$
$$-476M^4t^4/45D^8-16M^2t^6/3D^8-34t^8/45D^8]. \quad (9)$$

Finally, substitution of (9) in (1) yields the desired expression for the inductance of square tubular conductors.

$$L=W/8M^2t^2w^2=2.165260+4\log(D/M)-(t^2/3M^2)\log(M/t)-\pi t/3M-25t^2/36M^2+t^4/45M^4$$
$$+t^6/840M^6+t^8/1890M^8+M^4/10D^4+M^2t^2/3D^4+t^4/10D^4-17M^8/180D^8-2M^6t^2/3D^8$$
$$-119M^4t^4/90D^8-2M^2t^6/3D^8-17t^8/180D^8 \text{ abhenries per centimeter of bus length.} \quad (10)$$

The conductor spacings and dimensions used in practice are such that usually only certain terms of (10) are of consequence. Selecting these, we have

$$L=2.16526+4\log(D/M)-(t^2/3M^2)\log(M/t)$$
$$-\pi t/3M-25t^2/36M^2 \text{ abhenries per centimeter of bus length.} \quad (11)$$

The 60-cycle reactance, expressed in ohms per 1000 feet of conductor length, is obtained by multiplying L by 0.00574.

2. SOME APPLICATIONS

I. The following example, the data of which are taken from reference 1, illustrates application of (11).

Given: That $r=s=2.5$ inches; $t=0.5$ inch; $D=10$ inches. Hence, $M=2$ inches; whence by (11)

$$L=2.16526+6.43775-0.02888$$
$$-0.26180-0.04340$$
$$=8.26893 \text{ abhenries per centimeter of bus length.}$$

To the same number of decimal places (10) yields 8.26921; the difference of 0.00028, approximately 1 in 3000, is negligible. These results agree with the value 8.269 found in reference 1. The computation, however, requires less than three minutes, whereas that of reference 1 required more than an hour on an electric calculating machine.

II. Dwight and Wang[4] also have given formulas for the inductance and reactance of square tubular conductors. These are derived on the assumption that the conductors are so thin the sides can be considered as line segments, whence the geometric mean distances for such segments are then used to calculate the inductance. Such a procedure is equivalent to asserting that the current flows solely on the surface of the conductors; whence it follows from well-known theory that the values of inductance computed on this basis will be less than the actual values. Recognizing this restriction Dwight and Wang give additional formulas, empirically deduced, from which a correction for the effect of thickness can be calculated. As these results are presented in the form of curves quite convenient to use, it is of interest to check the accuracy of their expressions.

[4] H. B. Dwight and T. K. Wang, Trans. A.I.E.E. **57**, 762 (1938).

The reactances of typical busses comprised of standard square tubular conductors were calculated both from Dwight and Wang's formulas and from (10) above. Corresponding values are in substantial agreement. In general, by virtue

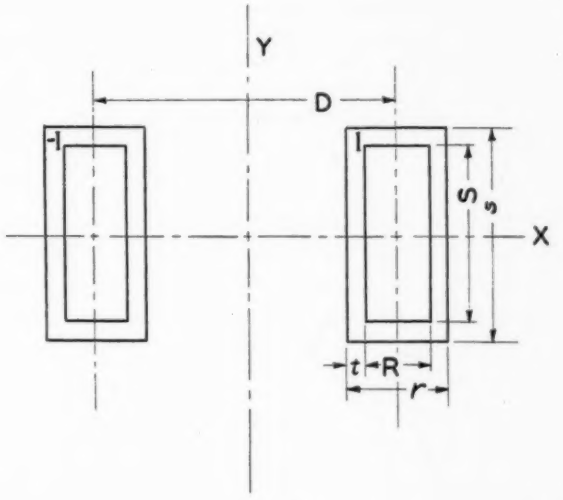

FIG. 1.

of the more rigorous treatment of the general problem, the writer's formula gives slightly higher values. Typically, for the data used above, the two values were 0.0469 and 0.0475 ohm per thousand feet of conductor length. Seemingly, for general direct-current and 60-cycle work either Dwight and Wang's or the writer's formulas yield sufficiently accurate values; for precise direct-current work, where skin and proximity effects are absent, the writer's formula yields slightly more accurate values; and for 60-cycle work, where eddy currents tend to reduce the inductance, Dwight and Wang's formula *may* be slightly more accurate.

If the edges of the square tubular conductor are rounded, the inductance is slightly increased. Dwight and Wang have given an empirical formula for correcting for this effect.

III. Finally, the inductance and reactance of polyphase circuits comprised of arrangements of parallel-sided square tubular conductors can be calculated by use of (10) or (11) in a manner described in detail in references 3 and 4.

Formulas for the Geometric Mean Distances of Rectangular Areas and of Line Segments

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Formulas are derived, by direct integration of the defining definite integrals, for the general geometric mean distance between (i) a point and a line segment; (ii) a point and a rectangular area; (iii) two parallel line segments; (iv) two orthogonal line segments; (v) a line segment and a parallel-sided rectangular area; (vi) two parallel-sided rectangular areas. Certain useful identities of the formula for the last-named case are presented.

IN central stations and in plants housing large units for electric welding, electric smelting, induction heating, electrolytic processing, or kindred operations, heavy-duty service busses are now constructed largely of conductors of structural shape. Thus, current designs of single and polyphase busses utilize rectangular straps, rectangular tubing, angles, channels, tees, and I-beams.

On the basis that the cross section of each of these types of conductors can be considered a combination of parallel-sided rectangular areas, the writer[1] has detailed a general method of calculating the inductance of a bus comprised of a group of these conductors and has given specific formulas for some of the more common types of busses. These formulas are expressed in terms of the geometric mean distances between the rectangles constituting the cross sections of the conductors. Often, though, the simpler postulate that the conductors are comprised of line segments affords, with much less computation, sufficiently accurate values of inductance. If so, the same formulas can yet be used, providing only that the geometric mean distances now be interpreted as those between line segments rather than rectangular areas. Regardless of the particular postulate adopted, however, use of these formulas requires *a priori* knowledge of the appropriate geometric mean distances.

In turn, calculation of these geometric mean distances usually requires knowledge of the formulas for the geometric mean distance between two *arbitrarily* located (i) parallel-sided rectangular areas; (ii) parallel line segments; or/and (iii) orthogonal line segments. *These formulas—seemingly, not to be found in the existent literature—are presented in this paper.*

1. DERIVATION FOR RECTANGULAR AREAS

The geometric mean distance D_{12} of a rectangular area S_1 from a second, arbitrarily located rectangular area S_2 is defined by

$$S_1S_2 \log D_{12}=\int_{S_2}\int_{S_1} \log [|X-x|^2+|Y-y|^2]^{\frac{1}{2}}dS_1dS_2; \tag{1}$$

(x, y) being the coordinates of a point in S_1 and (X, Y) those of a point in S_2. For convenience, the integrand of the right-hand member is designated hereafter by L.

If the rectangular areas are located as in Fig. 1, (1) yields

$$RSrs \log D_{12}=\int_{D+r}^{D+R+r}\int_{P+s}^{P+S+s}\int_0^r\int_0^s Ldydxd\,YdX. \tag{2}$$

Herein, $r \geq R$, $s \geq S$, and P and D take on negative as well as positive values. These selections detract in no whit from generality, yet enable all different positions of S_2 relative to S_1 to be encompassed within the first quadrant.

[1] T. J. Higgins, Trans. A.I.E.E. **61**, 53–57 (1943).

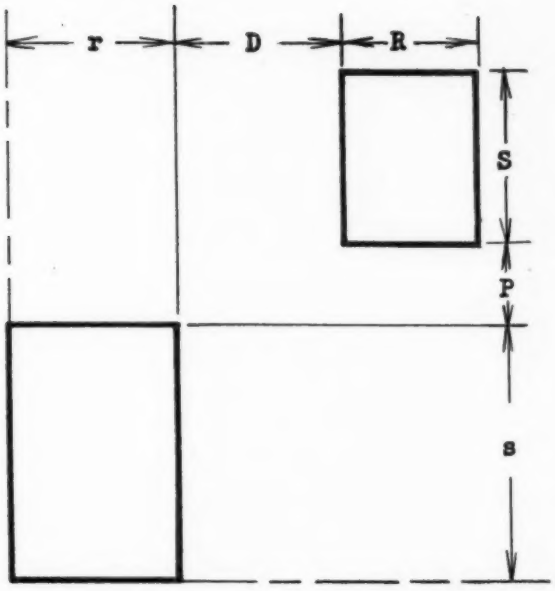

FIG. 1.

Explicit evaluation of the quadruple integral in (2) varies according as: (i) both P and D are positive; (ii) one or both of P and D are negative. In the first case, $X>x$ and $Y>y$; no logarithmic singularities are encountered in the course of evaluating the integral; and evaluation can be effected through direct use of known, tabulated integrals. In the second case, however, logarithmic singularities are encountered; the nature of these singularities varies with the relative position of S_1 and S_2; and evaluation is now complicated by the need of repeated use of limits and by the necessity of modifying the process of integration in accordance with the relative location of S_1 and S_2. This latter circumstance originates a formidable amount of rather elementary analysis: in fact, an amount so space filling, it is deemed advisable to present below only the end results of individual integrations; to reserve to the appendix tabulation of the indefinite integrals used to obtain these results; and to omit the intermediate analysis other than that demonstrated, illustratively, in the appendix.

Integrating first with respect to y, we distinguish two cases:

I(a). $X \geq x$, $Y \geq s$.

By (36) we obtain

$$\int_0^s L dy = -I_1[|X-x|, |Y-s|]+I_1[|X-x|, |Y|]-s; \tag{3}$$

wherein, typically,

$$I_1[|X-x|, |Y|]=|Y| \log [|X-x|^2+|Y|^2]^{\frac{1}{2}}+|X-x| \tan^{-1}[|Y|/|X-x|]. \tag{4}$$

I(b). $X \geq x$, $s \geq Y \geq 0$.

$$\int_0^s L dy = I_1[|X-x|, |Y-s|]+I_1[|X-x|, |Y|]-s. \tag{5}$$

Integrating secondly with respect to x, we distinguish four cases:

II(a). $X \geq x$; $Y \geq s$.

By (37) we have

$$\int_0^r \int_0^s L dy dx = I_2[|X-r|, |Y-s|]-I_2[|X-r|, |Y|]-I_2[|X|, |Y-s|]+I_2[|X|, |Y|]-3rs/2; \tag{6}$$

wherein, typically,

$$I_2[|X|,|Y|]=|X||Y|\log[|X|^2+|Y|^2]^{\frac{1}{2}}+\tfrac{1}{2}|X|^2\tan^{-1}[|Y|/|X|]+\tfrac{1}{2}|Y|^2\tan^{-1}[|X|/|Y|]. \quad (7)$$

II(b). $X\geq x$, $s\geq Y\geq 0$.

$$\int_0^r\int_0^s L\,dy\,dx=-I_2[|X-r|,|Y-s|]-I_2[|X-r|,|Y|]+I_2[|X|,|Y-s|]+I_2[|X|,|Y|]-3rs/2. \quad (8)$$

II(c). $r\geq X\geq 0$; $s\geq Y\geq 0$.

$$\int_0^r\int_0^s L\,dy\,dx=I_2[|X-r|,|Y-s|]+I_2[|X-r|,|Y|]+I_2[|X|,|Y-s|]+I_2[|X|,|Y|]-3rs/2. \quad (9)$$

II(d). $r\geq X\geq 0$; $Y\geq s$.

$$\int_0^r\int_0^s L\,dy\,dx=-I_2[|X-r|,|Y-s|]+I_2[|X-r|,|Y|]-I_2[|X|,|Y-s|]+I_2[|X|,|Y|]-3rs/2. \quad (10)$$

If, secondly, integration is performed with respect to Y rather than with respect to x—the integrand in (2) is such that the order of integration can be changed at will—we distinguish three cases. Integration yields, however, the same value in each case.

II′(a). $X\geq x$, $P\geq 0$;

II′(b). $X\geq x$, $P+S\leq 0$;

II′(c). $X\geq x$, $P+S>0>P$.

By (38) we find

$$\int_{P+s}^{P+S+s}\int_0^s L\,dy\,dY=I_2'[|X-x|,|P+S+s|]-I_2'[|X-x|,|P+S|]-I_2'[|X-x|,|P+s|]+I_2'[|X-x|,|P|]-3Ss/2; \quad (11)$$

wherein, typically,

$$I_2'[|X-x|,|P|]=-\tfrac{1}{2}[|X-x|^2-|P|^2]\log[|X-x|^2+|P|^2]^{\frac{1}{2}}+|X-x||P|\tan^{-1}[|P|/|X-x|]. \quad (12)$$

If, secondly, integration is performed with respect to X rather than with respect to either x or Y, we distinguish four cases.

II″(a). $D+r\geq x$, $Y\geq s$.

By (39) we obtain

$$\int_{D+r}^{D+R+r}\int_0^s L\,dy\,dX=-I_2''[|D+R+r-x|,|Y-s|]+I_2''[|D+r-x|,|Y-s|]+I_2''[|D+R+r-x|,|Y|]-I_2''[|D+r-x|,|Y|]-3Rs/2; \quad (13)$$

$$I_2''[|D+r-x|,|Y|]=|D+r-x||Y|\log[|D+r-x|^2+|Y|^2]^{\frac{1}{2}}+\tfrac{1}{2}|D+r-x|^2\tan^{-1}[|Y|/|D+r-x|]+\tfrac{1}{2}|Y|^2\tan^{-1}[|D+r-x|/|Y|]. \quad (14)$$

II″(b). $D+r\geq x,\ s\geq Y\geq 0.$

$$\int_{D+r}^{D+R+r}\int_0^s LdydX = I_2''[|D+R+r-x|,\ |Y-s|]-I_2''[|D+r-x|,\ |Y-s|]$$
$$+I_2''[|D+R+r-x|,\ |Y|]-I_2''[|D+r-x|,\ |Y|]-3Rs/2. \quad (15)$$

II″(c). $D+R+r\geq x\geq D+r,\ Y\geq s.$

$$\int_{D+r}^{D+R+r}\int_0^s LdydX = -I_2''[|D+R+r-x|,\ |Y-s|]-I_2''[|D+r-x|,\ |Y-s|]$$
$$+I_2''[|D+R+r-x|,\ |Y|]+I_2''[|D+r-x|,\ |Y|]-3Rs/2. \quad (16)$$

II″(d). $D+R+r\geq x\geq D+r,\ s\geq Y\geq 0.$

$$\int_{D+r}^{D+R+r}\int_0^s LdydX = I_2''[|D+R+r-x|,\ |Y-s|]+I_2''[|D+r-x|,\ |Y-s|]$$
$$+I_2''[|D+R+r-x|,\ |Y|]+I_2''[|D+r-x|,\ |Y|]-3Rs/2. \quad (17)$$

Reverting to the original schedule and integrating thirdly with respect to Y, we distinguish six cases: However, only two distinct values of the triple integral result therefrom.

III(a). $X\geq x,\ P\geq 0;$

III(b). $X\geq x,\ P+S\geq 0\geq P;$

III(c). $X\geq x,\ P+S\leq 0.$

By (40) we find

$$\int_{P+s}^{P+S+s}\int_0^r\int_0^s Ldydxd Y = -I_3[|x-r|,\ |P+S+s|]+I_3[|X-r|,\ |P+S|]+I_3[|X-r|,\ |P+s|]$$
$$-I_3[|X-r|,\ |P|]+I_3[|X|,\ |P+S+s|]-I_3[|X|,\ |P+S|]$$
$$-I_3[|X|,\ |P+s|]+I_3[|X|,\ |P|]-(11/6)Srs; \quad (18)$$

wherein, typically,

$$I_3[|X|,\ |P|]=[\tfrac{1}{2}|X||P|^2-(1/6)|X|^3]\log[|X|^2+|P|^2]^{\frac{1}{2}}$$
$$+\tfrac{1}{2}|X|^2|P|\tan^{-1}[|Y|/|X|]+(1/6)|Y|^3\tan^{-1}[|X|/|Y|]. \quad (19)$$

III(d). $r\geq X\geq 0,\ P\geq 0;$

III(e). $r\geq X\geq 0,\ P+S\geq 0\geq P;$

III(f). $r\geq X\geq 0,\ P+S\leq 0.$

$$\int_{P+s}^{P+S+s}\int_0^r\int_0^s Ldydxd Y = I_3[|X-r|,\ |P+S+s|]-I_3[|X-r|,\ |P+S|]-I_3[|X-r|,\ |P+s|]$$
$$+I_3[|X-r|,\ |P|]+I_3[|X|,\ |P+S+s|]-I_3[|X|,\ |P+S|]$$
$$-I_3[|X|,\ |P+s|]+I_3[|X|,\ |P|]-(11/6)Srs. \quad (20)$$

Integrating finally with respect to X, (41) yields the same value for all relative positions of S_1 and S_2. Accordingly, by (2) and (41) we obtain the desired formula.

$$\text{IV.} \qquad RSrs \log D_{12} = -(25/12)RSrs - (1/24) \sum_{i=1}^{4} \sum_{j=1}^{4} (-1)^{i+j} K(A_i, B_j); \tag{21}$$

wherein

$$K(A_i, B_j) = (A_i^4 - 6A_i^2B_j^2 + B_j^4) \log (A_i^2 + B_j^2)^{\frac{1}{2}} - 4A_iB_j^3 \tan^{-1} (A_i/B_j) - 4A_i^3B_j \tan^{-1} (B_j/A_i); \tag{22}$$

and

$$A_1 = |D+R+r|; \quad A_2 = |D+R|; \quad A_3 = |D|; \quad A_4 = |D+r|; \tag{23}$$

$$B_1 = |P+S+s|; \quad B_2 = |P+S|; \quad B_3 = |P|; \quad B_4 = |P+s|. \tag{24}$$

2. DERIVATION FOR LINE SEGMENTS

In the course of deriving, in Section 1, the formula for the geometric mean distance between two parallel-sided, rectangular areas, we have obtained expressions from which can be determined formulas for the geometric mean distance between all possible relative positions of (i) a point and a line segment; (ii) a point and a rectangular area; (iii) two mutually parallel or perpendicular line segments; (iv) a line segment and a parallel-sided rectangular area. We proceed to obtain these formulas.

By definition,

$$s^{-1} \int_0^s L dy$$

is the geometric mean distance between a point and a line segment. Two distinct formulas stem from I of Section 1.

By definition,

$$(rs)^{-1} \int_0^r \int_0^s L dx dy$$

is the geometric mean distance between a point and a rectangular area. Three distinct cases stem from II(a), II(b), and II(c). Insofar as relative position is concerned, II(d) is identical with II(b).

By definition,

$$(Ss)^{-1} \int_{P+s}^{P+S+s} \int_0^s L dy dY$$

is the geometric mean distance between two parallel (or collinear, if $x=X$) line segments. Only one formula stems from II′.

By definition,

$$(Rs)^{-1} \int_{D+r}^{D+R+r} \int_0^s L dy dx$$

is the geometric mean distance between two mutually perpendicular line segments. Four distinct formulas stem from II″.

By definition,

$$(Srs)^{-1} \int_{P+s}^{P+S+s} \int_0^r \int_0^s L dy dx dY$$

is the geometric mean distance between a line segment and a parallel-sided rectangular area. Two distinct formulas stem from III.

The final case, the geometric mean distance between two parallel-sided rectangular areas, has been identified already in IV.

Now, though none of these six geometric configurations has been systematically and completely studied hitherto, certain special cases of each configuration have been investigated. Corroboratively, then, we note that our formula stemming from case:

(1). I(b) agrees with Maxwell's;[2]
(2). II(c) agrees with Maxwell's;[2]
(3). II′(b) agrees with Maxwell's;[2]
(4). II″(a) agrees with Maxwell's;[2]
(5). III(c) yields Rosa's[3] corrected form of Gray's[4] formula for the geometric mean distance between a rectangular area and an externally and symmetrically located parallel line segment.
(6). IV yields Maxwell's[2] formula for the self-geometric mean distance of a rectangular area to itself; the writer's[5] formula for the geometric mean distance between two symmetrically superimposed, dimensionally different rectangular areas; Rosa's[3] corrected form of Gray's[4] formula for the geometric mean distance between two externally and symmetrically located, like or unlike rectangular areas; and Rosa's[3] formula for the geometric mean distance between two identical square areas, a side of each being collinear.

3. SOME USEFUL IDENTITIES

Through algebraic, geometric and trigonometric manipulations each of the formulas derived above can be transformed in various fashion. We give here some of the more important identities for the configuration of major interest, two parallel-sided rectangular areas. The following are easily established.

In general:

$$\begin{aligned} K(A_i, B_j) &= (A_i^4 - 6A_i^2B_j^2 + B_j^4)\log(A_i^2+B_j^2)^{\frac{1}{2}} - 4A_i^3B_j\tan^{-1}(B_j/A_i) - 4A_iB_j^3\tan^{-1}(A_i/B_j) \\ &= (A_i^4 - 6A_i^2B_j^2 + B_j^4)\log(A_i^2+B_j^2)^{\frac{1}{2}} + 4A_iB_j(A_i^2-B_j^2)\tan^{-1}(A_i/B_j) - 2\pi A_i^3B_j \\ &= (A_i^4 - 6A_i^2B_j^2 + B_j^4)\log(A_i^2+B_j^2)^{\frac{1}{2}} + 4A_iB_j(B_j^2-A_i^2)\tan^{-1}(B_j/A_i) - 2\pi A_iB_j^3. \end{aligned} \tag{25}$$

From (25) we obtain

$$K(A_i, B_j) = K(B_j, A_i); \tag{26}$$

$$K(A_i, A_i) = -2A_i^4(2\log A_i + \log 2 + \pi); \tag{27}$$

$$K(A_i, 0) = A_i^4 \log A_i; \tag{28}$$

$$K(0, B_j) = B_j^4 \log B_j; \tag{29}$$

$$K(0, 0) = 0. \tag{30}$$

If both P and D are positive, we have

$$2\pi A_i^3 \sum_{j=1}^{4} (-1)^j B_j = 2\pi B_j^3 \sum_{i=1}^{4} (-1)^i A_i = 0; \tag{31}$$

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[2] J. C. Maxwell, *A. Treatise on Electricity and Magnetism* (Clarendon Press, Oxford, England, 1892), third edition, p. 326.
[3] E. B. Rosa, Bull. Bur. Stand. **3**, 1 (1907).
[4] A. Gray, *Theory and Practice of Absolute Measurements in Electricity and Magnetism* (London, England, 1888–93), Vol. 2, part 1, p. 302.
[5] T. J. Higgins, Trans. A.I.E.E. **60**, 1046 (1941).

whence by (21), (25), and (31) we have

$$RSrs \log D_{12} = -(25/12)RSrs - (1/24)\sum_{i=1}^{4}\sum_{j=1}^{4}(-1)^{i+j}M(A_i, B_j); \tag{32}$$

wherein,

$$\begin{aligned} M(A_i, B_j) &= (A_i^4 - 6A_i^2B_j^2 + B_j^4)\log(A_i^2+B_j^2)^{\frac{1}{2}} + 4A_iB_j(A_i^2 - B_j^2)\tan^{-1}(A_i/B_j) \\ &= (A_i^4 - 6A_i^2B_j^2 + B_j^4)\log(A_i^2+B_j^2)^{\frac{1}{2}} + 4A_iB_j(B_j^2 - A_i^2)\tan^{-1}(B_j/A_i). \end{aligned} \tag{33}$$

From (33) we have

$$M(A_i, B_j) = M(B_j, A_i) - 2\pi A_iB_j(B_j^2 - A_i^2); \tag{34}$$

$$M(A_i, A_i) = -2A_i^4[2\log A_i + \log 2]; \tag{35}$$

$$M(A_i, 0) = M(0, A_i) = A_i^4 \log A_i. \tag{36}$$

APPENDIX

The elementary integrals required to effect the following integrations are to be found in any one of several well-known tables of indefinite integrals.[6] For compactness, we utilize the notation defined in Section 1. The significance of the $\pm$ notation is explained below.

$$\int Ldy = \pm\{I_1[|X-x|, |Y-y|] - |Y-y|\}. \tag{37}$$

$$\begin{aligned} \int\int Ldydx = \pm\pm\{&I_2[|X-x|, |Y-y|] - |X-x||Y-y|/2 + \pi|Y-y|^2/4\} \\ &+ \{\mp\pm x|Y-y|\}. \end{aligned} \tag{38}$$

$$\int\int LdydY = \pm\pm\{I_2'[|X-x|, |Y-y|] - 3|Y-y|^2/4\}. \tag{39}$$

$$\begin{aligned} \int\int LdydX = \pm\pm\{&I_2''[|X-x|, |Y-y|] - |X-x||Y-y|/2 + \pi|Y-y|^2/4\} \\ &+ \{\mp\pm X|Y-y|\}. \end{aligned} \tag{40}$$

$$\begin{aligned} \int\int\int LdydxdY = \pm\pm\pm\{&I_3[|X-x|, |Y-y|] - 5|X-x||Y-y|^2/12 \\ &+ \pi|Y-y|^2/12\} + \{\pm\mp\pm x|Y-y|^2/2\}. \end{aligned} \tag{41}$$

$$\begin{aligned} \int\int\int\int LdydxdYdX = \pm\pm\pm\pm\{&-(1/24)K[|X-x|, |Y-y|] - 13|X-x|^2|Y-y|^2/48 \\ &+ |X-x|^4/96 + \pi|X-x||Y-y|^3/12\} + \{\begin{matrix}+\\+\end{matrix}\pm\mp\pm xX|Y-y|^2/2\}. \end{aligned} \tag{42}$$

When using (37) to (42) inclusive to evaluate the definite integrals of Section 1, choice of the plus or the minus sign is determined as follows: if the upper or lower limit to be substituted for a particular variable is greater than the value (or values) of the other symbol within the absolute bars, the upper sign is chosen; if not, the lower sign is chosen. As an example illustrating both this procedure and the use of limits at a point of logarithmic singularity, we give in detail the evaluation of (5).

Here, $X \geq x$, $s \geq Y \geq 0$; whence the integrand, $L \equiv \log[|X-x|^2 + |Y-y|^2]^{\frac{1}{2}}$, of the integral

$$\int_0^s Ldy$$

[6] H. B. Dwight, *Tables of Integrals and Other Mathematical Data* (Macmillan Company, New York, 1934).

has a logarithmic singularity at $y=Y$. In consequence, we have by (37)

$$\int_0^s Ldy=\lim_{\epsilon\to 0}\int_0^{Y-\epsilon} Ldy+\lim_{\epsilon\to 0}\int_{Y+\epsilon}^s Ldy$$

$$=-\{\lim_{\epsilon\to 0}[|\epsilon|\log[|X-x|^2+|\epsilon|^2]^{\frac{1}{2}}+|X-x|\tan^{-1}[|\epsilon|/|X-x|]-|\epsilon|]$$

$$-[|Y|\log[|X-x|^2+|Y|^2]^{\frac{1}{2}}+|X-x|\tan^{-1}[|Y|/|X-x|]-|Y|]\}$$

$$+\{[|Y-s|\log[|X-x|^2+|Y-s|^2]^{\frac{1}{2}}+|X-x|\tan^{-1}[|Y-s|/|X-x|]-|Y-s|]$$

$$-\lim_{\epsilon\to 0}[|\epsilon|\log[|X-x|^2+|\epsilon|^2]^{\frac{1}{2}}+|X-x|\tan^{-1}[|\epsilon|/|X-x|]-|\epsilon|]\};$$

the minus sign in (37) being used in evaluating the first integral since both of the limits, 0 and $Y-\epsilon$, are less than Y; while the plus sign in (37) is used in evaluating the second integral since both of the limits, $Y+\epsilon$ and s, are greater than Y. Passing to the limit yields (5):

$$\int_0^s Ldy=I_1[|X-x|,|Y-s|]+I_1[|X-x|,|Y|]-s.$$

Calendar of Meetings

April

2– 3 American Physical Society, Southeastern Section, Auburn, Alabama
2– 3 American Physical Society, New York State Section, Rochester, New York
3 American Physical Society, New England Section, Wellesley, Massachusetts
8– 9 American Institute of Electrical Engineers, Pittsfield, Massachusetts
18–24 American Ceramic Society, Pittsburgh Pennsylvania
22–24 American Philosophical Society, Philadelphia, Pennsylvania
23–24 American Geophysical Union, Washington, D. C.
23–24 American Mathematical Society, New York, New York
23–24 American Mathematical Society, Chicago, Illinois
24 American Mathematical Society, Stanford University, California
26–28 American Society of Mechanical Engineers, Davenport, Iowa
28–30 American Institute of Electrical Engineers, Kansas City, Missouri
29–30 American Institute of Mining and Metallurgical Engineers, Cleveland, Ohio

May

14–15 Acoustical Society of America, New York, New York

June

14–16 American Society of Mechanical Engineers, Los Angeles, California
14–19 American Association for the Advancement of Science, Corvallis, Oregon
14–19 American Association of Physics Teachers, Corvallis, Oregon
16–17 American Mathematical Society, Corvallis, Oregon
18–20 American Association of Physics Teachers, Chicago, Illinois
18–20 Society for the Promotion of Engineering Education, Chicago, Illinois
21–25 American Institute of Electrical Engineers, Cleveland, Ohio
28–July 2 American Society for Testing Materials, Pittsburgh, Pennsylvania

Letter to the Editor

Proposal for the Extension of Our System of Mensuration

EUGENE C. BINGHAM
Lafayette College, Easton, Pennsylvania

October 25, 1942

SOME time ago there arose a need for a means of exhibiting visually the relations between the various units of length in the English and metric systems and the various distances in the universe. At first it appeared impossible thus to compare microscopic distances such as the Angstrom, micron, mils with the distances reached by the telescope, e.g., the astronomical unit, the light year, and parsec. However, it is practicable to make all of these lengths, Table I, visually comparable as shown in Fig. 1.

TABLE I.

			Length in cm	qD
Metric				
Micromicron			1×10^{-10}	$\bar{10}.0000$
Angstrom unit			1×10^{-8}	$\bar{8}.0000$
Millimicron			1×10^{-7}	$\bar{7}.0000$
Micron			1×10^{-4}	$\bar{4}.0000$
Millimeter			0.1	$\bar{1}.0000$
Centimeter			1.0	0.0000
Decimeter			10.0	1.0000
Meter			100.0	2.0000
Kilometer			1×10^{5}	5.0000
English				
Mils	0.001	in.	0.00254	$\bar{3}.40483$
Point (printing)	0.01389	in.	0.03528	$\bar{2}.54753$
Em (printing)	1/6	in.	0.4233	$\bar{1}.62665$
Inch			2.54	1.40481
Hand	4	in.	10.16	1.00689
Span	9	in.	22.86	1.35908
Link (surveyor's)	0.66	ft.	20.1168	1.30356
Foot	12	in.	30.48	1.48401
Yard	3	ft.	91.44	1.96144
Rod (surveyor's)	16.5	ft.	502.92	2.70150
Mile (statute)	5280	ft.	160,934	5.20664
Mile (nautical)	6080.2	ft.	185,325	5.26793
Astronomical unit (earth to sun)			1.495×10^{13}	13.17464
Light year			9.4637×10^{17}	17.97606
Parsec			3.084×10^{18}	18.48911
qD Values of Certain Distances				
X-ray wave-length of uranium K series				-8.8981
Radius of Bohr orbit of normal hydrogen, 5.281×10^{-9} cm				-8.27701
Distance between atoms in rocksalt crystal, 2.814×10^{-8} cm				-7.5507
Red cadmium spectrum line at 15° C, $6.438{,}469{,}6\times10^{-5}$ cm				-4.19122
Height of Washington Monument, 555 feet				4.2283
Height of Eiffel tower, 3.00×10^{4} cm				4.4771
Distance from New York to Philadelphia, 83 miles, 1.335×10^{7} cm				7.1255
Distance from New York to San Francisco, 2568 miles, 4.132×10^{8} cm				8.6162
Diameter of the earth, 7927 miles, 1.275×10^{9} cm				9.1055
Circumference of the earth, 24,900 miles, 4.005×10^{9} cm				9.6026
Distance traveled by light in one second $2.997{,}76\times10^{10}$ cm				10.4768
Diameter of the sun, 864,000 miles				11.1429
Distance of planet Pluto, 41.5 astron. units				14.7927
Distance to nearest fixed star, α-Centaurus				18.6096
Diameter of the galaxy, 10^{5} light years				22.976
Distance of Andromeda nebula, 720,000 light years				23.833

When we are required to compare objects, we naturally do this relatively rather than absolutely. Thus we say that A is twice or three times as long as B. To state that A is two "sizes" larger than B is more difficult because it requires the knowledge of how large the size unit appears at the same distance away. The importance of the ratio instead of the absolute length is not novel and underlies Fechner's law of grading. If lengths differing from each other by a constant ratio, e.g., 10, are arranged in order of increasing sizes of magnitude, the relation between the *lengths* and *sizes* is logarithmic,

$$y=10^{x}$$

where y is the length and x the corresponding "size." It is obvious that the exponent is here a Briggsian logarithm, and it has the important property of varying much less rapidly than the length, and the dimension is equally well defined by either x or y. It so happens that, although we have a plethora of units of length, we have no name for this complementary unit. We may as well admit that most persons are not accustomed to thinking in terms of logarithms, but that is hardly a valid objection for there are some who do not use the multiplication table.

The physical chemists, being faced with the above problem in connection with wide ranges of ionic concentration, have avoided the above difficulty. They have given to $-x$ a name of its own, pH, which is sufficiently distinctive so that it is not misunderstood. We will make use of the same idea with one modification. We will let qD be the logarithm of the length expressed in centimeters. Since the values of qD for all lengths less than one centimeter are negative, we may write $pD=-qD$, pD being the logarithm of the reciprocal of the lengths. The conception of qD as a true unit of dimension has many advantages. The most important one is that a multiplicity of units is unnecessary. Conversion from one system to another involves a single addition or subtraction, e.g., from inches to centimeters by the addition of 0.40483. World uniformity is obtained by the use of the centimeter as standard, but the base 10 is the important matter. No distance yet measured in the universe requires a figure higher than $25qD$, and microscopic dimensions within the atom do not go to $20pD$, hence the use of large numbers of zeros before or after the decimal point are avoided. For example, the distance traveled by light in one second is 29,977,600,000 cm, an important constant of nature, often written $2.997{,}76\times10^{10}$ cm. Either method of expression is cumbersome because we have to think not only of the significant figures, but of the number of zeros and the unit employed. It is expressed with equal accuracy as 10.476795 qD, or approximately 10.48 qD, the characteristic giving at once the order of the distance, while the mantissa gives the desired number of digits. Conversions from qD values to lengths may, of course, be made with logarithmic tables, but, where lengths are used in calculations involving multiplication or division, the qD values are more convenient. Thus, for example, since a mile has a qD value of 5.20664, the distance in miles which light travels in one second is $10.476795-5.270145$, i.e. 186,170. This would make it desirable to have published the logarithms of important constants of nature, but that is already being done to a certain extent.

The subject of sizes and grades of manufactured articles and products is a rather difficult one, because it is natural to start out with a constant increment, yet as the object

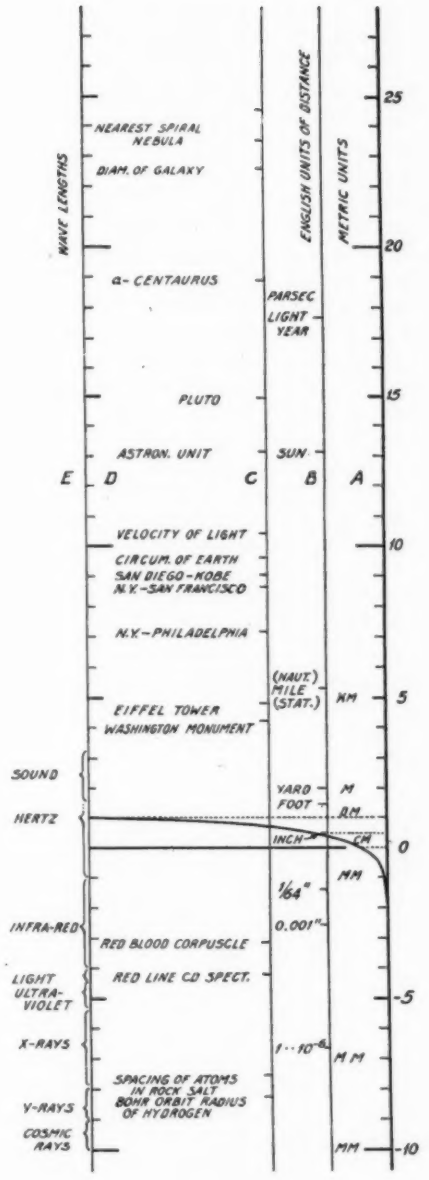


FIG. 1

measured becomes larger, the system must sooner or later break down, for the increment then becomes inappreciable and therefore entirely too small. Conversely, in going to the very small sizes or grades the decrements become much too large. This is not very important in the objects like hats, shoes, and other apparel where the range of sizes is small, but with mechanical materials, rods, wire, screws, etc., the case is different. There have been many gauge sizes proposed for general use, e.g. the Birmingham, Stub, and Brown & Sharpe gauge "numbers" for wire. All gauges take advantage of the truth that the increments should increase as the size of the object measured is increased, but not all of them recognize the importance of having the ratio between the successive sizes constant, and perhaps none recognizes the importance of using the above equation, making y equal to the standard unit of length, when $x=0$. Thus the wire gauges are impractical in the sense that tables are required for conversion. The qD values are not arbitrary or rounded off, and they are a true measure of length without any conversion whatever.

By plotting Browne and Sharpe numbers, Table II, Fig. 2, against the corresponding lengths in inches, one will observe that a hyperbolic curve is obtained. If, however, these same numbers are plotted against the qD values of the lengths, a straight line is obtained which would be perfectly linear had not "rounding off" been employed. If sizes start with zero, for unit length, one obtains a series of sizes which may be given as shown in Table III. The sizes are qD values, the lengths repeat themselves with

TABLE II.*

B. & S. Gauge No.	Diameter in inches	Log Diam.	Diff.
10	0.102	−0.5866	
			0.0496
11	0.091	−0.6362	
			0.0505
12	0.081	−0.6867	
			0.0512
13	0.072	−0.7379	
			0.0511
14	0.064	−0.7890	
			0.0503
15	0.057	−0.8393	
			0.0483
16	0.051	−0.8876	

* The differences between the logarithms of the diameters given in the last column are nearly equal as they would be in a qD method of sizing. A difference of 0.0500 corresponds to a ratio of 1.12, or 1¼. If the B. & S. numbers given here are represented by x and the corresponding lengths by y, we find that

$$\log y = -0.0817 - 0.05046x = (9.9183 - 10) - 0.05046x$$

which gives 0.326 in. when $x=0$ and is not a recognized standard of length. The relation proposed here, *viz.*, $\log y = qD$, is simpler and therefore preferable and it does give the standard unit of length when the size is zero.

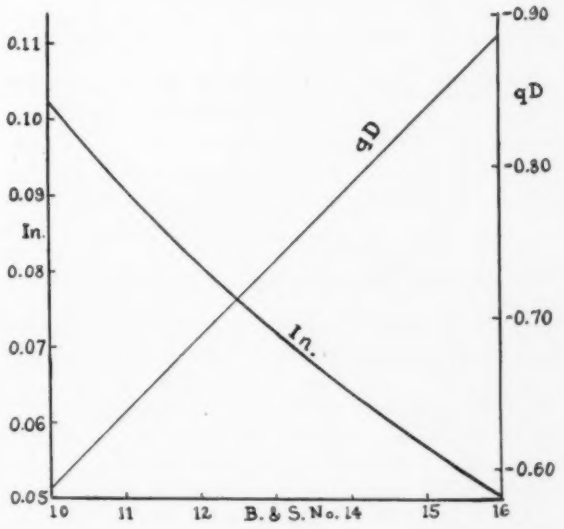


FIG. 2.

TABLE III

Size No.	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Lengths	1.0	1.6	2.5	4.0	6.3	10	16	25	40	63	100

each increase of a full size, multiplied by ten. These lengths are the antilogarithms of the equally spaced qD values, and they are obviously "rounded off" values, as may be seen from Table IV. The important thing to observe is

TABLE IV.

qD	Corresponding exact length*
0.0	1.0000
0.2	1.5849
0.4	2.5119
0.6	3.9811
0.8	6.3097
1.0	10.0000

* The numbers in the second column are the exact values of the so-called "preferred numbers." The qD values are however simpler and form an arithmetical series.

merely that $16/10=25/16=40/25=63/40$ approximately. European and American writers[1] have referred to the numbers 1, 1.6, 2.5, etc., as "preferred numbers," but these rather badly rounded-off numbers[2] do not strike one as in any way to be preferred. The amount of the rounding-off is indicated by Table IV. It is merely the equal spacing of the qD values that makes them important, and there are many other arithmetical series of qD values that might by employed, for example, such as 0, 0.1, 0.2, 0.3, etc., with the ratio of 1.2589 instead of 1.585. In manufacture and trade the choice of the ratio becomes of importance.

The writer has suggested that glues, pitches, and other products of rheological interest be graded on the above system. Taking the unit of fluidity, $\varphi=1$ rhe, as the standard, zero grade would correspond to one rhe, $q\Phi=0$; the fluidity of water at 20°, which is 100 rhes, would then have a $q\Phi=2.00$. The fluidity of castor oil is 0.10 rhe at 20°, hence $p\Phi=1.0=-q\Phi$. In covering the range from hydrogen gas, $\Phi=11{,}760$, to hard glass at room temperature, with $\Phi=10^{-19}$, there are some twenty-three full grades or 115 subdivisions if each full grade is divided into five, as suggested. Peter Cooper grades of glue are about thirty in number; twenty grades of pitch have been suggested.

In Fig. 1, the qD scale is at the bottom, extending from -10 to 28 units of qD, divided decimally. On the scale above that, the unit lengths of the metric system are indicated and on the B scale the most used units of the English system. On scale C are noted a few distances of interest, some particularly well defined. The D scale is added for convenience in reference, and the various wavelength bands are indicated on scale E.

Three lengths are drawn to scale in the figure as ordinates, *viz.*, the centimeter (taken as standard), the inch, $(0.40\ qD)$, and the decimeter $(1\ qD)$, all indicated by dotted lines. The millimeter $(-1\ qD)$ is also visible although not dotted. The line joining the upper ends of these lengths gives the portion of the logarithmic curve, of which the part from -1 to 0 is identical with that from 0 to 1, etc., except that the ordinates are tenfold greater for the latter. It should be noted that the qD value for 0.2 cm is $\bar{1}.301$ if the mantissa is read in the positive direction from left to right or as 0.699 when read as a pD value.

It is obvious that by extending the idea here advanced, an entire qX system of units can be evolved, using not only length, fluidity, but mass, time, energy, etc., as well. It is of interest to note that mass and energy scales turn out to be longer than the length or times scales, but it is noted that a volume scale would be three times as long as the qD scale.

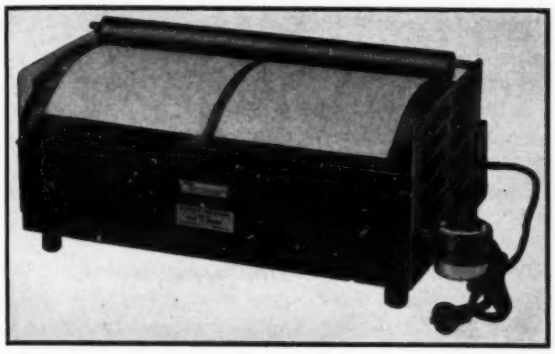
There are certain advantages of the use of qD values which are not new, as they have long been appreciated in connection with preferred numbers, but those advantages have not been fully utilized. When similar machines are made in different sizes, it is a great convenience if the draughtsman can simply add the same increment to all of the dimensions. Where gear wheels, pulleys, shafting, bolts, screws, and innumerable parts are involved, it is essential that the parts be available in stock. If all parts of machines are designed with a system of qD values which have been agreed upon parts will correspond and much of the expensive administrative costs of production can be dispensed with. The number of sizes demanded must always be a matter for agreement.

[1] Hirshfeld and Berry, Mech. Eng. **44**, 791 (1922) and many others.
[2] E.g., 64 instead of 63.

Innovations in Instruments

Spee-Dee Printer

A Spee-Dee Printer for making blueprints or black and white prints in half-a-minute has been developed by Peck and Harvey, Chicago. This portable, table model can be plugged into any standard electric outlet. It comes in two sizes: for prints up to 12×18 and 18×24. The actual printing surface is over size on both size machines. This prevents the crowding of prints, and overlapping or blurred edges. Printing on a curved glass surface gives perfect contact without the inconvenience of wrapping the prints around a cylinder. You can duplicate tracings,



drawings, letters, invoices, or any written or printed matter on the Spee-Dee. It enables you to reproduce drawings immediately after they leave the drafting table. The prints made on the Spee-Dee cost less than 1½¢ per square foot. For prices and further details, write to Peck and Harvey, 4325 Addison Street, Chicago, Illinois.

Three New Insulating Alternates for Varnished Silk

Varnished rayon, varnished cotton cloth, and varnished nylon have been developed by the Irvington Varnish & Insulator Company for electrical insulation formerly provided by varnished silk. These materials possess good dielectric strength with tensile and tear strengths equal to or better than varnished silk and in addition can be punched into special shapes. They are available in thicknesses from 0.003 inch to 0.008 inch in straight-cut rolls or bias-cut strips in 51-inch lengths. Each base material is coated with insulating varnish.

High tenacity varnished rayon is the most suitable alternate for varnished silk, comparing favorably with it in strength and flexibility. It has a dielectric strength of 1200 vpm and is used for wrapping leads, small magnetos, and coils.

Varnished cotton cloth has greater tensile strength than varnished silk. Its pliability permits application on odd shapes. Dielectric strength is 1200 vpm.

Varnished nylon has qualities of flexibility and high tensile strength with dielectric strength of 1200 vpm. At this time, nylon is only available by government allocation.

New Combination Insulator and Wire Marker

Short lengths of extruded plastic tubing which are marked with letters and numerals serve as insulators of terminal connections and as wire markers. Where lug insulation and wire identification are required, they speed assembly by eliminating any additional means of identification.

The tubing from which these new combination insulator-markers are made has very high dielectric strength. Smooth inside surfaces permit quick application over wires and lugs. Legible numerals of the user's choice are printed on the tubing with an ink that has resistance to chemicals, water, and oils equal to that of the tubing itself. Available in colored tubing with either black or yellow symbols, in A.S.T.M. sizes from No. 9 up to ⅜ inches I.D. Larger sizes may be made upon special order.

Other tubing made from much less critical plastic materials can be printed and utilized solely as wire markers. Manufactured by the Irvington Varnish & Insulator Company, 6 Argyle Terrace, Irvington, New Jersey.

New Development in Ultraviolet-Ray Irradiation

A new "merry-go-round" system of group ultraviolet-ray irradiation, adapting tested British methods for reducing absenteeism due to sickness, is announced by Dr. Charles Engelhard, President of the Hanovia Chemical and Manufacturing Company, Newark, New Jersey, for use in American war plants.

"To receive applications workers stand on a circular platform which revolves around a special high pressure quartz health lamp supported by a pedestal type column. This lamp is so set that the ultraviolet rays are equally distributed to each of eight to ten people standing on the platform. Rotation of the platform is automatically timed to give each person on it the regular five-minute prophylactic treatment approved by medical authorities."

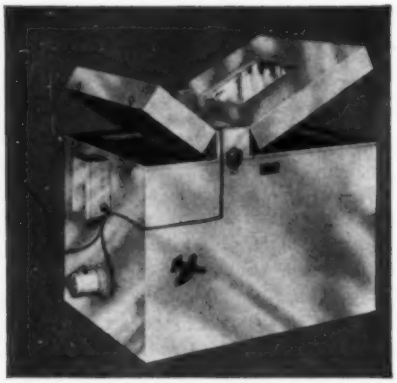
This first merry-go-round measures 14 feet in diameter and is of sufficient size to provide treatments to 100 workers an hour. The platform is geared to make 12 revolutions an hour, and 8 to 10 employees ride for each treatment.

"So far this country has lagged far behind the British in the application of artificial sunlight in industrial hygiene," the company states. "While individual health lamps are installed in many of our plants, there are yet no systematic methods of group treatment in operation as they are in England."

New Constant Temperature Dry-Ice Cabinet

A new development for use where refrigeration is required in the laboratory is offered by the American Instrument Company, Silver Spring, Maryland. The cabinet provides temperatures from minus 90 F up to 220 F, with a constancy of plus or minus ½ F. It is intended for use where expensive mechanical refrigeration would not be justified.

Two temperature ranges are available: from zero to minus 90 F, and from 220 F to minus 90 F. Temperatures of minus 65 F and 90 F are attained in 15 and 30 minutes,



respectively, from an initial temperature of 85 F. Temperatures can be held at minus 40 F and minus 90 F in an ambient temperature of 85 F for 24 hours with 40 and 60 lb. of dry ice, respectively.

The cabinet is portable and ready for operation after packing with dry ice and plugging the cord into the current supply. Once packed with dry ice it requires no further attention until a new charge of dry ice is required.

The temperature within the cabinet is held constant within plus or minus ½ F. In the low temperature model, this close temperature control is made possible by means of an Aminco Quickset Bimetal thermo-regulator, which, through a solenoid and an electronic relay (time-delay), operates a damper that allows air to be passed over the dry ice when cooling is needed, or to be by-passed when cooling is not needed. The temperature control system requires only natural heat leakage for its operation.

New Cathode-Ray Tubes

The RCA Manufacturing Company, Inc. is now making available to equipment manufacturers the following new cathode-ray tubes for use in connection with WPB rated orders:

RCA-3BPI is a 3-inch, high vacuum, cathode-ray tube having electrostatic deflection, electrostatic focusing, green fluorescence, and medium persistence. It has a 2-inch diameter bulb neck, separate leads to all deflecting electrodes and the cathode, and an over-all length of about 10 inches. All leads terminate in the diheptal base which permits wide separation of the low voltage pins from the high voltage pins.

RCA-3EP1/1806-P1 is a high vacuum tube similar to the 3BPI. It has the same ratings as the 3BP1 but a different bulb with 1¾-inch diameter neck and a magnal base. Separate leads to all deflecting electrodes are provided, but the cathode is connected to the heater within the tube.

RCA-7CP1/1811-P1 is a short 7-inch, high vacuum, cathode-ray tube having magnetic deflection, electrostatic focusing, green fluorescence, and medium persistence. It has a 1¾-inch diameter bulb neck and an over-all length of about 13½ inches. Except for anode No. 2 which is connected to a snap terminal on the side of the bulb, the other electrodes, including the cathode, all have separate leads terminating in an octal base.

Additional information on these three new cathode-ray tubes may be obtained from RCA Manufacturing Company, Inc., Harrison, New Jersey.

Mercury Laboratory Arc

The Gates Mercury Laboratory Arc employs a General Electric high intensity mercury burner, consisting of a fused quartz arc tube enclosed in a hard glass outer bulb. The entire unit is self-contained and the burner is conveniently mounted on an adjustable standard which permits placement in almost any position on the standard, or removal for remote use at the end of an extension cord.

This equipment provides the line spectrum of the high pressure mercury arc from the infra-red to the limit of transmission of Pyrex glass in the near ultraviolet. It is particularly well adapted for optical slit illumination, due to its high brightness, cool operation, and steady illumination over long periods of time. It also is useful as a source of monochromatic light in optical work, because of the brightness and purity of its easily seen mercury-green line at 5461 angstrom units. The impurity of this band is so low that it may be used for interferometry.

The blue line at 4539 angstroms is also a powerful source of monochromatic radiation in this region. The 10140 infra-red line is valuable as a reference point for work in

this part of the spectrum where calibration is particularly difficult.

A folder describing this lamp, its characteristics, and chief uses will be furnished upon request by Geo. W. Gates & Company, Franklin Square, Long Island, New York.

New Pressure and Detonation Pick-Ups

The Electro Products Laboratories offer new electro pressure and detonation pick-ups to be used with Diesel and gas engines, also pumps, compressors, and similar mechanisms to record instantaneous pressures and other phenomena occurring within the firing or compression chambers.

It consists of a magnetic pick-up, having a diaphragm, which is exposed to the explosion or pressure forces within the cylinder; the vibration of this diaphragm produces magnetic flux variations in a coil assembly which provides an output voltage having identical characteristics to the varying pressures developed in the cylinder.

Two types are now available, namely, Models No. 3000 and No. 3000A. The Model No. 3000 fits into a hole in the cylinder which is tapped to receive a $\frac{7}{8}$-inch 18 thread. The No. 3000A fits into the standard aircraft engine 18-mm spark plug hole.

Output voltages as high as 40 millivolts are obtainable from these pick-ups when used with the standard C. F. R. gasoline testing engine using 70-octane rating gasoline, this voltage being the total of detonation and regular explosion. Should detonation or any cyclic engine pressure voltage be

required, the Electro Filter Model No. 2300 can be used to select these voltages over a range from 2000 to 8000 cycles.

These pick-ups are designed for operation at excessive temperatures met with in gas engines because of the high temperature insulation over the fine windings of this pick-up, also proper ventilation. Special construction of this pick-up eliminates binding post connections. It is only necessary to screw on the socket attached to a shielded 7-ft. cable to the end of the pick-up and then a completely shielded connection is automatically made to the pick-up terminals. The complete electrical shielding of the pick-up and cable eliminates ignition and other undesired electrical interference.

Two types of diaphragms are furnished with each pick-up, one for ordinary pressures met with in gas engines and compressors and the other for high compression aviation engines.

New Instrument Booklets

"Electronics—A New Science for a New World" is the name of a booklet issued by General Electric presenting the story of electronics—its past, its present, and its great possibilities for the future. "Electronics," states the booklet, "is the science of the electron—a tiny, invisible particle of pure electricity, the basis of all familiar matter. Only within the last two generations has science discovered how to control electrons by the vacuum tube, and put them to work for the good of mankind." This booklet, which has many colored illustrations, tells how the electron is working today in war combat; in research; in industry to step up production, increase human efficiency, and reduce material waste; in radio and television to extend the range and quality of sound and sight over the air waves; in agriculture to improve quantity and quality; and in medicine to reveal more of the structure and behavior of the human body. A copy of the booklet may be obtained by writing the General Electric Company, Schenectady, New York, and asking for booklet GED-1024 (31 pages).

The Leeds & Northrup Company have just put out two new catalogs, "Type U Test Set" and "Type S Test Set." They revise considerably their earlier publications describing this equipment.

"Type U Test Set" is of special interest to telephone and other communication engineers. This portable Wheatstone bridge is designed especially for communication purposes in measuring resistance and capacitance. It is best adapted for locating faults on telephone and telegraph cable, for identifying faulty wires in a cable, for measuring conductor resistance, for locating grounds and crosses by Murray, Varley, and Hilborn loop tests, and for locating opens by capacitance tests (Catalog E-53-441(1), 8 pages).

"Type S Test Set" has been designed for manufacturers of wire, motor windings, electrical instruments and meters, and electrical household appliances, as well as many laboratories, repair shops, and maintenance departments, where routine resistance tests are made. It is a general-purpose Wheatstone bridge with self-contained galvanometer and battery. This test set is simple in arrangement, and portable (Catalog E-53-400(1), 8 pages).

To obtain these catalogs write to Leeds & Northrup Company, 4934 Stenton Avenue, Philadelphia, Pennsylvania, stating the name and catalog number.

The *Ohmite News* of December, 1942 had short articles on "Important Features of Ohmite Rheostats," "Effect of

Enclosure on Resistor Temperature," "Resistor Cages for Safety and Unit Production." The January, 1943 *Ohmite News* tells of "Important Features of Ohmite High Current Tap Switches," "Fundamental Ohm's Law," and "Ohmite 2½ Meter R.F. Plate Choke in Wide Use." By addressing the Ohmite Manufacturing Company, 4835-41 Flournoy Street, Chicago, Illinois, you can obtain Catalog 18 which gives the Ohmite stock free of charge, and an Ohm's Law Calculator for ten cents.

The International Nickel Company, Inc., 67 Wall Street, New York, New York, offers five new publications free of charge.

Heat Transfer Through Metallic Walls contains charts and formulas with an explanation of the method for calculating the over-all heat transfer rates of different metals and alloys in actual service.

The Working of High-Nickel Alloys gives instructions for hot forming, cold forming, shearing, punching, deep drawing, spinning, heat treating, pickling, welding, soldering, riveting, silver brazing, and all machining operations. It includes reference data on compositions, properties, and typical applications for 17 different materials. (Reprinted from American Machinist.)

Relaxation Resistance of Nickel-Alloy Springs reports the investigation on the load-carrying ability of nickel-alloy springs at temperatures ranging from 300 to 700 F. (Reprint of the report by the Special Committee on Mechanical Springs of the American Society of Mechanical Engineers.)

Index to Specifications for INCO Nickel Alloys gives all specifications issued by the United States Government, the American Society for Testing Materials, the American Society of Mechanical Engineers, and the Aeronautical Materials Specifications.

Engineering Properties of "K" Monel is a revised bulletin on "K" Monel and "KR" Monel (improved machinability).

The booklet entitled "Micromax Thermocouple Pyrometers" (Catalog N-33A) gives war-time pyrometer users information about available instruments—indicators, recorders, and controllers—and about the thermocouples and accessories which are used with them. There is a description, for the new or prospective user of balance-type (potentiometer) pyrometers, of the application of the basically-sound balance method to the measurement and control of thermocouple temperatures. The present book differs from the 1941 edition only in minor details, but it is extensively revised compared to the 1939 and previous editions; an automatically-balanced Micromax Indicator and the Model C Controller (non-indicating) for flush mounting have been added, the section on thermocouples has been expanded, and automatic control is outlined more fully. A copy of the 1942 edition may be obtained from Leeds and Northrup Company, 4934 Stenton Avenue, Philadelphia, Pennsylvania (58 pages).

"Electric Resistance Measurements," D-1, besides listing the 638-1 Kelvin Wheatstone Bridge and 621 A percent bridge which were in the previous D bulletin, features the

630 Wheatstone Bridge. The 630 Wheatstone Bridge is an electrical measuring instrument suitable for laboratory and manufacturer for production testing. Manufactured by the Shallcross Manufacturing Company, Collingdale, Pennsylvania.

The 1942 edition of the "RCA Guide for Transmitting Tubes" is completely revised and much new material has been added. This guide is organized into three major sections. The first section is entitled "Transmitting Tube Data." It deals with basic circuits and socket connections for popular power tubes. Gas triodes, gas tetrodes, and the u-h-f acorn and midget types are included. There is also a special reference chart cataloging air-cooled and water-cooled transmitting tubes, transmitting and television rectifiers, cathode-ray tubes, photo-tubes, voltage-regulator tubes, and special purpose tubes. The second section, "Transmitting-Circuit Facts," includes information on the design, adjustment, and operation of transmitters. The third section, "Transmitter Construction," describes the layout, adjustment, and operation of equipment in several rigs. A copy may be obtained from local RCA distributors or by sending thirty-five cents to Commercial Engineering Section, RCA Manufacturing Company, Inc., Harrison, New Jersey (72 pages).